Cointegration between Fama-French Factors

Abstract

Cointegration has many applications in finance and other fields of science researching time series and their interdependences. The analysis is a useful method to analyse non-cointegration time series, which in this study are Fama-French factors. If the previous relationships can be formed to a stationary linear combination, the previous series are cointegrated. FF-factors are factors, which affect the share's yield expectations in the long term i.e. each factor has a premium of different size over the risk-free interest. In this study we use cointegration to find dependencies between different factors. We also study the balances and dynamics between prices, after which we create an error correction model based on 4 hedge portfolios.

Keywords: cointegration, three-factor model, Johansen procedure, hedging

1. Introduction

The cointegration analysis became an important part of econometrics shortly after it was published (Engle, Granger 1987). It has traditionally been applied broadly to a wide variety of time series, when one has desired to investigate interrelationships of those series. The method is thus useful in many sectors of science, but in particular it has attracted interest in the financing, where it probably has been applied to study the relationships between different local markets. The idea is simply to find common stochastic trends between the series. The method is intended to examine non-stationary time series, as are usually the prices of securities. Compared with the traditional correlation analysis, cointegration allows creating a model to forecast the time series. Cointegration also demands investigated time series to be reverting towards the mean. When series diverge, they may be strongly correlating, though not necessarily cointegrated. Referring to the previous, the correlation analysis may fit better the short-term perspective, while the cointegration can be used in both short-term and long-term dynamic studies (Alexander 1999).

In this study, the examination focuses analysing the Fama-French three-factor model (Fama, French 1996) with the cointegration of the market portfolio. The purpose is to examine the price balances and the dynamics of the earnings. Moreover, we also must pay attention to the long-term trends of the series It is possible to apply an adjustment term when the series' drifts are different, which term must be used between the FF-factors, as these factors are partly diverging from each other. The starting assumption is that the markets and the FF-factors are alone non-stationary, but that it is possible to generate a stationary linear combination between those series. Non-stationary means in this study a random walk process, while the assumption between the factors is that they don't behave completely "random walk". Thus together series can wander anywhere, but not alone. Previous examination responds quite well with the cointegration between the index and the single stock (e.g. Alexander 1999).

First, the FF factors average yields above the risk-free interest rate are compared to Russell's style investment indices for comparable yields over the risk-free interest. Then the differences between the indices and factors are analysed. In this case the differences are the size of the share and the ratio between the book value and market value (BE/ME). Then an ADF test (Dickey-Fuller 1979) decides whether the selected indices are non-stationary, otherwise the cointegration analysis cannot be applied. Then the cointegration between market portfolio and the indices is analysed with the CVAR model (cointegrated vector autoregressive model), and when cointegration is found the error correction model ECM is used to make predictions. Now the multiple-time series cointegration review must be done with the Johansen procedure (Johansen 1988; Johansen, Juselius 1990). Finally, 4-style investment hedge portfolios are built for different training periods based on ECM's forecast properties. After that these strategies are compared with the market portfolio and index returns.

2. Fama-French three-factor model

The Fama-French three-factor model (Fama, French 1993, 1996) is currently the best and most widely used model for "anomalies" (Cochrane 1999), which CAPM (Sharpe 1964; Lintner 1965) cannot explain to. These "anomalies" count the share size and book-to-market BE/ME-value’s effect on the long-term expected returns. The shares seem to have a strong value premium a (high-BE/ME-value), which has been observed in empirical studies (Rosenberg, Reid, Lanstein 1985), which in turn demonstrates that the value shares’ returns are significantly higher than the growth shares’. In addition, small stocks have been found to produce higher returns than large stocks in the long-term period.

In the three-factor model, expected returns depend on the market risk b_i and the risk-free interest rate R_f and in addition, share size and BE/ME-value also affect expected returns. The model follows the equation

$$E(R_i) = R_f + b_i(E(R_M) - R_f) + s_i E(\mathrm{SMB}) + h_i E(\mathrm{HML}), \tag{1}$$

Where E (R_i) is the expected return on the chosen shares i and E (R_M) is the expected return on the whole market portfolio. SMB is the difference between the expected returns on small and large shares. Correspondingly HML is the difference between expected returns on high-BE/ME and low-BE/ME. Factor weights b_i, s_i and h_i can be determined from portfolio components in a simple linear regression. In our study, the market is divided into 9 portfolios B/L, B/M, B/H, M/L, M/M, M/H, S/L, S/M, S/H, where the first character tells the portfolio’s share size $\mathrm{big} = \mathrm{B}$, $\mathrm{mid} = \mathrm{M}$ and $\mathrm{small} = \mathrm{S}$ and the second character tells BE/ME value $\mathrm{high} = \mathrm{H}$, $\mathrm{medium} = \mathrm{M}$ and $\mathrm{low} = \mathrm{L}$. One factor is always divided into three parts, where a single component represents 33% of the number of shares. SMB and HMB consist of the following equations (2) and (3)

$$\mathrm{SMB} = (\mathrm{S/L} + \mathrm{S/M} + \mathrm{S/H})/3 - (\mathrm{B/L} + \mathrm{B/M} + \mathrm{B/H})/3 \tag{2}$$

$$\mathrm{HML} = (\mathrm{S/H} + \mathrm{M/H} + \mathrm{B/H})/3 - (\mathrm{S/L} + \mathrm{M/L} + \mathrm{B/L})/3\,. \tag{3}$$

The previous two FF factors are very low correlated 0.13 (Davis, Fama, French 2000). In the same article, the previous 9 portfolios’ yields were studied in the U.S. market in the time interval 1929 - 1997, in which 339 shares of NYSE were used over the period 1929-1952, since in the year of 1953 the number of NYSE shares had been doubled. In the year of 1996 the number of shares was 4,562 in NYSE, AMEX and Nasdaq stock markets. Moreover the model has been tested in other major capital markets. The FF-factor model's return differences (Davis, Fama, French 2000) have been reported in Table 1. As the risk-free interest rate R_f the United States 1-month treasure bill is used.

StockSymbol	BE/ME	Size(millions)	Extra returns (R_i-R_f)(%)/annum	Volatility(σ)(%)
B/L	0.43	94.7	7.19	
B/M	1.04	92.1	8.99	
B/H	1.87	89.5	12.68	
M/L	0.53	55.9	8.73	
M/M	1.07	55.1	12.01	
M/H	2.18	53.2	14.43	
S/L	0.55	22.4	7.57	
S/M	1.11	22.2	13.35	
S/H	2.83	19.1	15.94	
Russell 3000	market portfolio		8.30	15.9
Russell 1000 growth	large cap growth		7.58	20.2
Russell 1000 value	large cap value		9.22	14.0
Russell 2000 growth	small cap growth		5.11	22.3
Russell 2000 value	small cap value		10.63	17.6

Table 1. Fama-French factors versus Russell indices and their extra returns/annum and volatilities.

In later cointegration analyses, the factors studied are Russell's five indices (Russell 2006) in 1980 - 2004. The market portfolio is described by the Russell 3000 Index, which covers approximately 98% of the U.S. stock market's value. As a large cap portfolio is used the Russell 1000 growth index, which corresponds to the B/L-portfolio and the other large cap portfolio is the Russell 1000 value index, which in turn corresponds to the B/H portfolio. Single shares to the previous indices have been selected in the Russell 1000 index based on the BE/ME values and the Russell 1000 index contains the 1,000 largest shares. In the same ways as the small cap portfolio is used the Russell 2000 growth index, which corresponds to the S/L-portfolio and as the other small cap portfolio is the Russell 2000 value index, which in turn corresponds to the S/H portfolio. Single shares to previous indices have been selected in the Russell 2000 index based on the BE/ME values and the Russell 2000 index containing the 2,000 smallest shares.

The portfolios, which Fama and French formulated, correspond quite well to Russell's indices (Table 1). Value shares, which have a high BE/ME value, have clearly higher average return than growth shares, but the size impact is not so that clear. However, it has been observed that in the longer term the small-value shares produce higher returns than larger shares. This is clearly seen in (Table 1) and (Figure 1), where value shares have given better returns between the years 1980 - 2004. On the other hand the growth-shares show contradictory results, because large growth shares have surprisingly returned more during the years 1980 - 2004 than smaller, though in the years 1929 - 1997 the situation (Davis, Fama, French 2000) has been the opposite. The results are nevertheless quite contradictory comparing to the CAPM model, because during the years

1980 - 2004 the higher volatility growth stocks have given the lowest returns compared to other Russell indices.

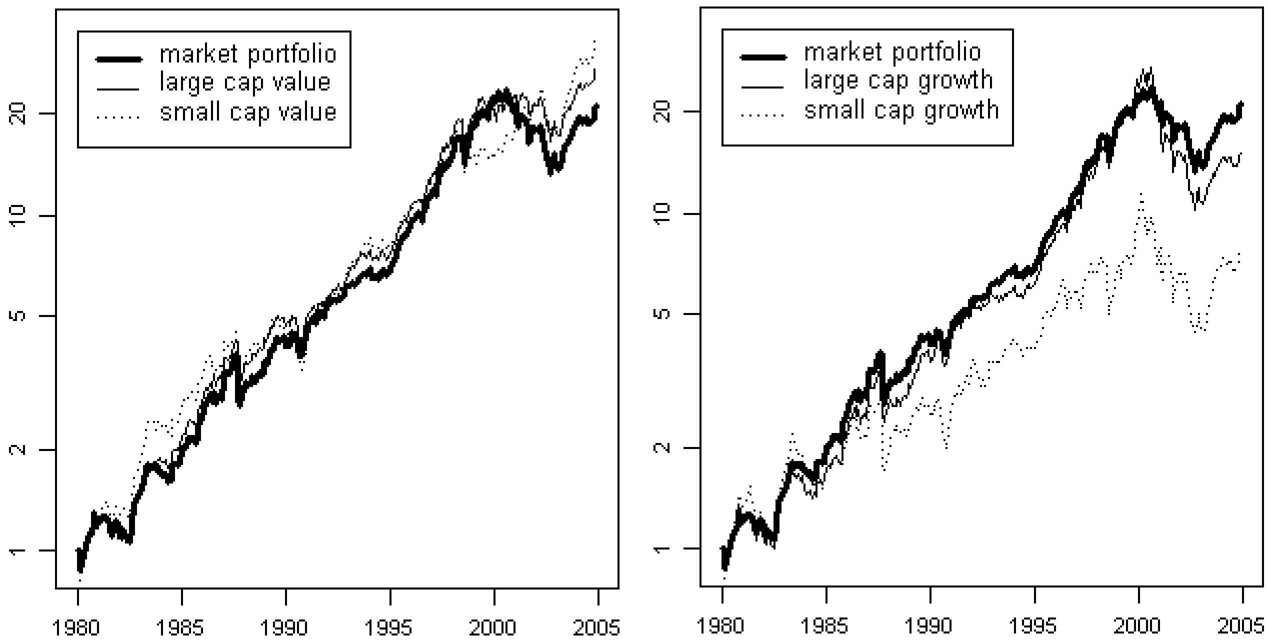

Figure1. Normalized prices of Russell indices.

3 Cointegration

Cointegration between time series is one of the most important econometric tools, which has been used widely, since the Engle-Granger two-step method appeared (Engle, Granger 1987, the Nobel Committee 2003). Cointegration can be used to obtain important information of the time series' long-term structure, which then can be used to improve the economic decision-making. The previous cointegration either exists or not (on/off-principle). A good result can be achieved only by using carefully statistical analysis, leaving still a small probability to fail. Two or more non-stationary time series, which are integrated to a degree of $I(n)$, can represent linear combinations, where they are stationary. Thus these series are cointegrated $I(0)$.

In this review series are integrated to the degree $I(1)$ (non-stationary) or they are not integrated $I(0)$ (stationary). If the series x_t and y_t are integrated to a degree $I(1)$, but their linear combination

$$y_t = a + bx_t + \varepsilon_t \tag{4}$$

is I(0), the series x_t and y_t are cointegrated and the error term ε_t is in the form

$$z_t (= \varepsilon_t) = y_t - a + bx_t \sim I(0) \tag{5}$$

being stationary, so that a and b exist and a is a possible drift vector. Now the vector z_t is called a cointegration vector, which properties are tested later in this study. If there exist a number of n series, there cannot be more than (n-1) cointegration vectors z_t. If there exist only two time-series, there can thus be only one vector, because otherwise the original series should be stationary (Alexander 1999).

One of the most important features of the cointegrated series is their common stochastic trend (Stock, Watson, 1988). The series x_t and y_t are thus linked to each other in a long-term period. These series may be separated in the short term, but in long term they follow suit, which is called "long-run equilibrium". If the series diverge without limit and no correction term is used, these series do not have a common balance relationship and thus cointegration does not exist. A stochastic trend can be presented for two series as follows

$$x_t = \mu_{xt} + \varepsilon_{xt}, \tag{6}$$

$$y_t = \mu_{yt} + \varepsilon_{yt}, \tag{7}$$

where μ_{xt} and μ_{yt} are averages of the series x_t and y_t, which depend on previous averages and their related error terms. ε_{xt} and ε_{yt} are distances from the averages. Now that x_t and y_t are cointegrated, they can be presented as a linear combination

$$b_1 y_t + b_2 x_t = (b_1 \mu_{yt} + b_2 \mu_{xt}) + b_1 \varepsilon_{yt} + b_2 \varepsilon_{xt}, \tag{8}$$

where $c = (b_1 \mu_{yt} + b_2 \mu_{xt})$ must be stationary. The coefficients b_1 and b_2 can be solved with e.g. linear regression. The series x_t and y_t may now be presented in the form

$$x_t = \mu_{xt} + \varepsilon_{xt}, \tag{9}$$

$$y_t = -\frac{b_2}{b_1}\mu_{xt} + \frac{c}{b_1} + \varepsilon_{yt}, \tag{10}$$

because they have a common stochastic trend and then they are thus cointegrated.

In the following chapters 3.1-3.2, the previous theory is explained based on the Johansen procedure and later it is used to create a simple hedging strategy.

3.1 Johansen procedure

Using the Johansen procedure in cointegration analysis can be applied more than just two time-series (Johansen 1988; Johansen, Juselius 1990), therefore the procedure has become a main tool in cointegration analysis. We use it also in this study, because the Engle-Granger method cannot be used for the five time-series situation. The Johansen procedure is based on finding a stochastic matrix eigenvalues, which will also help to reduce the correlation related problems. The biggest difference from the Engle-Granger method is to focus maximum stationarity instead of the minimum variance principle. Furthermore, the test is more versatile and sophisticated compared to the Engle-Granger method, but correspondingly more complex (Alexander 1999). Cointegration can also be found divergenting from series, if the trend corrective term μ is used, which is required when investigating FF-factors, because the returns differ in the long term.

First we must create an n-degree and p-dimensional long-term VAR model (vector autoregressive model) (11), from which to create a cointegration basic model.

$$y_t = \Pi_1 y_{t-1} + \ldots + \Pi_n y_{t-n} + \mu + \Psi D_t + \varepsilon_t \tag{11}$$

y_t is now the process vector $(p \times 1)$ at the time t, in which at least two components must be non-stationary. ε_t $(p \times 1)$ is the error-term vector, where the errors are independent of each other. Π_i $(p \times p)$ is the process' y_{t-i} coefficient matrix at the time t_i, while D_t is the vector of non-stochastic variables, such as the dummy variables for which the Ψ is the coefficient matrix. μ in turn is the unlimited drift, which takes into account the different sized drifts from the observed time-series. Non-stochastic effects vectors don't appear in our case, so now $D_t = 0$. The next CVAR model (cointegrated vector autoregressive model) is built on equation (11)

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_n \Delta y_{t-n+1} + \Pi y_{t-n} + \mu + \varepsilon_t, \tag{12}$$

where

$$\Gamma_i = -(1 - \Pi_1 - \ldots - \Pi_i), \text{ where } i = 1, \ldots, n\text{-}1 \tag{13}$$

$$\Pi = -(1 - \Pi_1 - \ldots - \Pi_n) \tag{14}$$

and $\Delta y_i = y_i - y_{i-1}$. Apart from the VAR model the review is now focused on the matrix Π rank(Π) degree, which will tell the most essential information of the series' long-term relationships. Now the term Πy_{t-n} must be stationary $I(0)$. If matrix Π degree is $\text{rank}(\Pi) = p$, the matrix Π ισ is then a full degree matrix and all the components of y_t are stationary, when the initial assumptions of non-stationarity cannot be revised. If also $\text{rank}(\Pi) = 0$, is Π null matrix and consequently the model is no longer a CVAR model. When the matrix Π degree is $1 < \text{rank}(\Pi) < (1 - p)$, there exist cointegration vectors and the matrix Π can be represented in the form $\Pi = \alpha\beta^T$, where α and β are full degree matrices. α describes a long-term adjustment speed and β in turn describes the cointegration vectors. The cointegration hypothesis $H_0(r)$ is of the form

$$H_0(r) : \Pi = \alpha\beta^T, \tag{15}$$

in which case the process Δy_t is stationary and according to the initial assumptions of the series y_t at least two of previous series are non-stationary.

The previous estimation of the model begins with maximum likelihood procedure, which initially begins by finding the Gaussian errors in the multivariate cointegration model (Johansen 1988, 1991; Johansen, Juselius 1990). The likelihood function parameters $\Gamma_1, \ldots, \Gamma_{n-1}$ and μ must be defined using regression with the Δy_t and y_{t-n} utilizing the terms $\Delta y_{t-1}, \ldots, \Delta y_{t-n+1}$. This gives the residuals R_{0t} and R_{nt}, which can be used to determine the cross moment matrix residual's S_{ij}

$$S_{ij} = T^{-1}\sum_{t=1}^{T} R_{it}R_{jt}^{T}, \quad i,j = 0,n, \tag{16}$$

where T is the time matrix from null to time T. The centralized likelihood function is

$$R_{0t} = \alpha\beta^{T} R_{nt} + \varepsilon, \tag{17}$$

where ε represents the error. The regression equation (17) can be used to estimate α as a function of β

$$\hat{\alpha} = S_{0n}\beta(\beta^{T} S_{nn}\beta)^{-1}, \tag{18}$$

after which β can be determined by solving eigenvalues λ_i from the equation (19).

$$\left|\lambda S_{nn} - S_{n0}S_{00}^{-1}S_{0n}\right| = 0 \tag{19}$$

Eigenvalue problem (19) has now p solutions, $1 > \hat{\lambda}_1 > ... > \hat{\lambda}_p > 0$. Corresponding eigenvectors are found as $\hat{V} = (\hat{v}_1,...,\hat{v}_p)$ and they can be represented in a normalized form

$$\hat{\beta} = (\hat{v}_1,...,\hat{v}_r), \tag{20}$$

where $\hat{\beta}$ is the maximum likelihood estimate and it is also given by $r = \text{rank}(\Pi)$. The maximum likelihood function has the form

$$L_{\max} = \left|S_{00}\right|\prod_{i=1}^{r}(1-\hat{\lambda}_i). \tag{21}$$

Next the likelihood ratio test is carried out for the hypotheses (15) in the equation (11) VAR model (Johansen, Juselius 1990). There exist two different LR-tests, of which the first one is the Trace statistic (22) and the second is the λ_{max} statistic (23). The tests solve the relevant roots or eigenvalues, whereby the rank of the matrix Π can be decided.

$$LR_{trace} = -T\sum_{i=r+1}^{p}\ln(1-\hat{\lambda}_i) \tag{22}$$

$$LR_{\max} = -T\ln(1-\hat{\lambda}_{r+1}) \tag{23}$$

In the LR-test the null hypothesis is applied H_0: $\lambda_{r+1} = \lambda_{r+2} = ... = 0$, which gives the system p-r unit roots, which are the starting point for finding system rank. The roots are found step by step, where first is assumed that there exist p unit roots. If the null hypothesis H_0 has to be rejected, the answer is $\lambda_1 > 0$, after which the hypothesis H_0: $\lambda_2 = \lambda_3 = ... = \lambda_p = 0$ is applied. If this again is rejected, the result is $\lambda_2 > 0$, after which the process is repeated all the way to p, unless the unit root can be found. If the hypothesis is finally accepted, the amount of cointegration vectors are found by using the unit roots. The last described rank of the matrix Π is the most important and difficult part the of Johansen procedure. If the rank is estimated too low, the cointegration may be unnoticed. A too large degree of rank can lead to discovering a cointegration, though it does not really exist.

3.2 Results

Before cointegration can be tested, the original time series logarithms' stationarity must be tested. To realize stationarity, the coefficients of the time-series term γ (equation 24) must have a smaller absolute value than 1. In stationary series the shocks are temporary and they will always return slowly to their average level. Non-stationary series typically do not have a long-term equilibrium, like stocks and indices in general. If the series is non-stationary, there is always a unit root.

For roots exploration, there are many tests. However, in this study we use the most widely known augmented Dickey-Fuller test (ADF-test) (Dickey-Fuller 1979). The tested time series y_t is now given by

$$y_t = a + \gamma y_{t-1} + \gamma_1 \Delta y_{t-1} + ... + \gamma_p \Delta y_{t-n} + \varepsilon_t , \qquad (24)$$

where a is a constant (drift), γ is the coefficient of lag change Δy and n is the lag degree in the autoregressive process. Next is tested the market portfolio's and FF factor's stationarity, which are now solely created from the Russell indices. As null hypothesis H_0 is applied $\gamma = 1$, thus the series is non-stationary and there exists a unit root. As alternative hypothesis is applied H_1, thus the series is stationary. Previous test results have been gathered in Table 2. with lags = 1-5 and it will be noticed from them that all portfolios are non-stationary $H_0(p>0.05)$ for each five lags.

Portfolio name	lag = 1		lag = 2		lag = 3		lag = 4		lag = 5	
	Dickey-Fuller	*p*-value	Dickey-Fuller	*p*-value	Dickey-Fuller	*p*-value	Dickey-Fuller	*p*-value	Dickey-Fuller	*p*-value
market portfolio	-1.848	0.640	-1.745	0.684	-1.795	0.662	-1.748	0.682	-1.821	0.651
large cap growth	-1.656	0.721	-1.672	0.714	-1.874	0.629	-1.985	0.582	-2.102	0.533
large cap value	-1.545	0.768	-1.493	0.790	-1.431	0.815	-1.302	0.870	-1.426	0.818
small cap growth	-3.238	0.082	-3.171	0.093	-2.804	0.237	-2.859	0.214	-2.439	0.391
small cap value	-0.128	0.990	-0.136	0.990	-0.023	0.990	0.430	0.990	0.145	0.990

Table 2. Statistical significance of non-stationary *p*-value.

Complete the next (logarithmic) error analysis for indices using the equation (4), where a and b can be defined with a linear regression (for example OLS regression). In this situation the error term ε_t does not need to be noticed. Analysing errors, matrix Π rank is assumed to be $r = p = 5$. The time-series error correlations (ρ-*residual*) and their standard deviations are listed in Table 3. Between some errors, there are significant correlations, as for example between the market and large cap growth portfolios, where the correlation is as high as 0.967, while the correlation between the large cap value and the small cap growth is only 0.723. The normality of errors has further been tested with the Shenton-Bowman test (Shenton, Bowman 1977; Doornik, Hansen 1994), where normality or autocorrelation in errors do not occur, when lags is raised to six. Autocorrelation, however, can be found, if the used lag is too small.

ρ-*residual*	market portfolio	Large cap growth	large cap value	small cap growth	small cap value
market portfolio	1				
large cap growth	0.967	1			
large cap value	0.937	0.827	1		
small cap growth	0.868	0.860	0.723	1	
small cap value	0.863	0.768	0.843	0.895	1
	standard deviations of the residuals				
	0.0174	0.0201	0.0159	0.0272	0.0187

Table 3. Correlation matrix and standard deviations of the residuals.

The next step is to test the existence of the long-term cointegration between market portfolio and FF factors 1980-2004 (Hansen, Juselius 1995). First all the five time-series must undergo the trace statistic estimation according to the equation (22). In Table 4., there are the previous test *p*-values of the null hypothesis H_0. In Table 4. can be found three different unit roots, which are $\lambda_3 = \lambda_4 = \lambda_5 = 0$, $\lambda_1 > 0$ and $\lambda_2 > 0$. The matrix Π rank is tentatively 2, so there are also 2 cointegration vectors.

Hypothesis	$r = 0$	$r \le 1$	$r \le 2$	$r \le 3$	$r \le 4$
p-value	0.000	0.043	0.272	0.166	0.198

Table 4. Johansen Trace statistics *p*-values.

The matrix Π rank r can be further verified by using the companion matrix *A* eigenvalues (Hansen, Juselius 1995),

$$A = \begin{bmatrix} \Pi_1 & \Pi_2 & \cdots & \Pi_{n-1} & \Pi_n \\ I_p & 0 & \cdots & 0 & 0 \\ 0 & I_p & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & I_p & 0 \end{bmatrix} \tag{25}$$

where I_p is a *p*-dimensional identity matrix and Π_i is defined in the equation (11). 30 eigenvalues of the matrix *A* are described in the Picture 2. unit circle. The eigenvalues must be located at the unit circle or inside it, unless the matrix Π rank cannot be 2. In our situation all the eigenvalues are, however, at the unit circle or the inside it, so the matrix Π rank is 2. Thereby it has also 2 cointegration vectors β, which reflect the market risk for each factor and market portfolio.

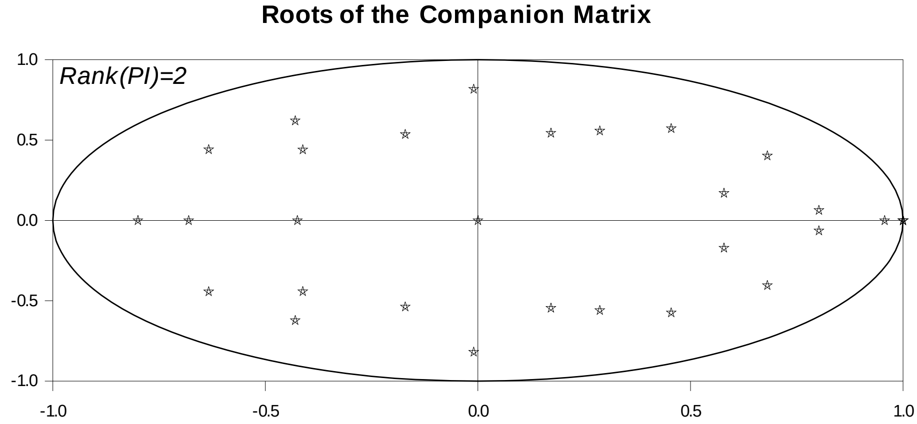

Figure 2. A scatter plot of the eigenvalues of the companion matrix.

Matrix Π has now got an estimate $\hat{\Pi}$ and so have all the other CVAR equation (12) parameters been estimated. Thus, the matrix Π has been resolved, after which can be build error correction model ECM for forecast. The previous review has also been represented separately in Table 5. between all single time-series. In combination of the two time-series, there cannot be found cointegrations except of a pair of large cap value and small cap value.

lag = 6	market portfolio	large cap growth	large cap value	small cap growth	small cap value
market portfolio	-				
large cap growth	0	-			
large cap value	0	0	-		
small cap growth	0	0	0	-	
small cap value	0	0	1	0	-

Table 5. Rank of cointegration matrix Π between single time-series.

Finally, using the error correction model, there has been created simple hedging strategy, where the target investments are the 4 FF-factors based on the Russell indices. The hedge portfolio weights are updated every three months, so that the factor, which the ECM model predicts to grow, the highest return receives a portfolio weight 1 and the other factors' weights are 0. The weights are presented every quarter in Appendix 1. Four different training periods are used to test the predictions. The used training periods are 5-year, 7-year, 10-year and all previous data, and the predictions are for the period 1991Q1 - 2004Q3. Also in the initial scenario used period 1980Q1 – 1990Q4 has a matrix Π, which rank is 2, so it is rational to use two cointegration vectors, at every stage.

All 4 training periods' cumulative nominal returns are illustrated in Figure 3. The studied 14 year period cointegration hedging strategy has won a market portfolio (Russell 3000 index). Returns are also compared apart from the market portfolio to the best index small cap value returns. In the Figure 3. cases b), c) and d), where a 7 years or longer training period has been used for a 3 month prediction prior to its starting point, the hedging strategy was able to win even the high return small cap value index (Russell 2000 value). On the other hand a 5-year training strategy was unable to win the best index, though it won the market portfolio. In Table 6., the market portfolio, factors and hedge portfolios' returns over risk-free interest, are shown together as has been done in Table 1. as also their most important descriptive statistics for the 14-year period.

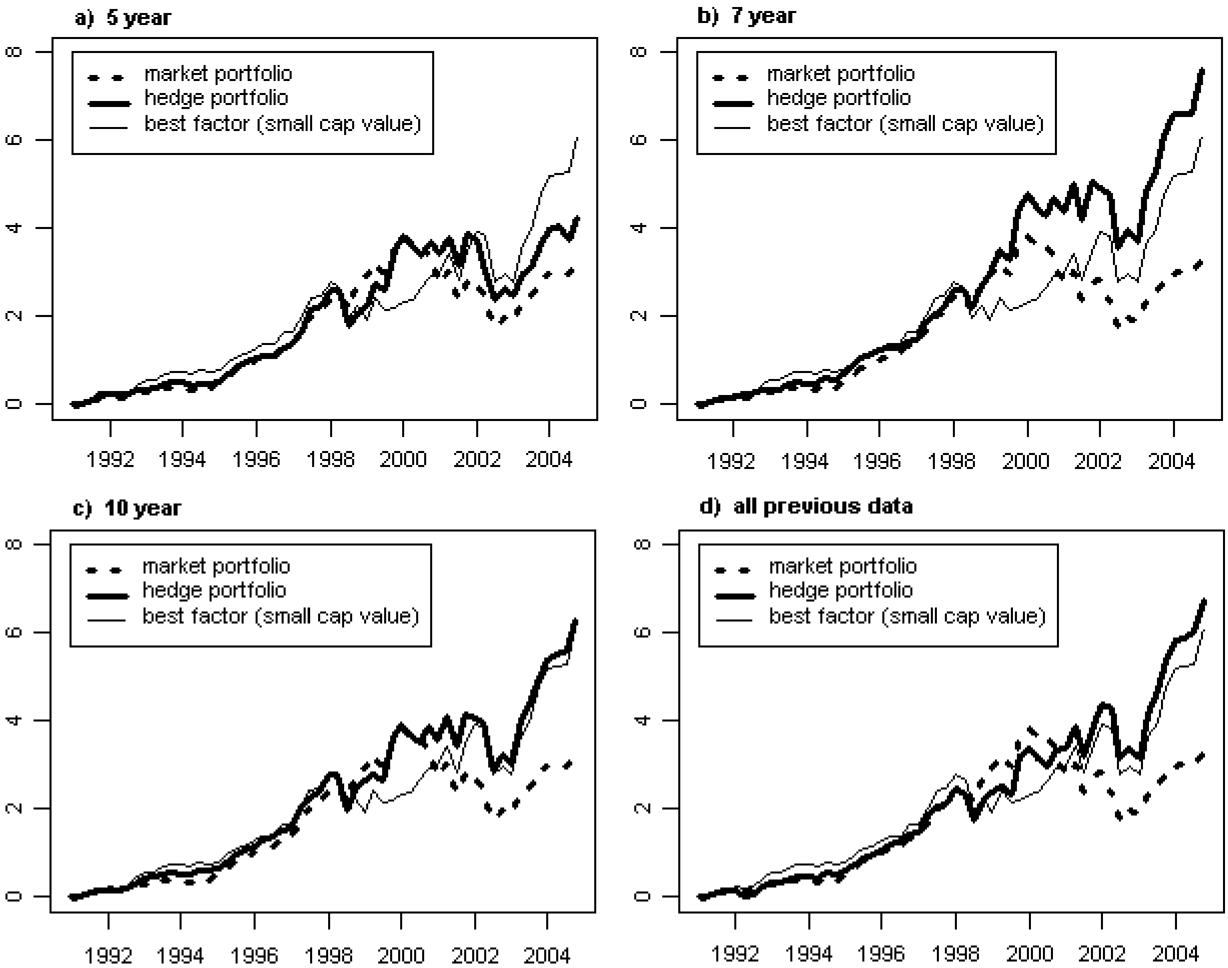

Figure 3. Cumulative nominal returns of 5,7,10 years and all previous data hedge portfolios.

	Extra returns(R_i-R_f)(%)	Volatility(σ)(%)	Sharpe ratio	Alpha(%)
market portfolio	9.59	15.8	0.60	0
large cap growth	8.37	19.9	0.41	-1.22
large cap value	10.88	14.1	0.76	1.28
small cap growth	6.61	25.4	0.26	-2.98
small cap value	13.73	16.6	0.82	4.13
hedge portfolio (5 year)	11.57	17.7	0.65	1.97
hedge portfolio (7 year)	15.53	16.5	0.94	5.93
hedge portfolio (10 year)	13.83	17.4	0.79	4.23
hedge portfolio (all prev. data)	14.12	17.4	0.80	4.52

Table 6. Portfolios' descriptive numbers/annum(%) 1991Q1 - 2004Q3.

According to Figure 1. and 3. the differences between the indices have been greatest around the turn of the millennium, when the IT bubble has made growth stocks unreasonably expensive. Precisely in that period of hedging strategy has performed best. In Figure 3. strategies b), c) and d) has exploited really well the unusual high returns of the growth stocks in the late 1990s, and after that it managed to jump off the ride changing to value stocks, when the growth stocks have become disproportionately expensive. This is very understandable, because the value stocks were rising then quite moderately and the cointegration expects the series to be mean reversing. When the gap grew too high, the model simply jumped out of the growth shares.

Cointegration has been studied much between the market location factors and the properties of single stock the index (e.g. Alexander 1999), but cointegration can also be examined between the FF-factors, which in this study was able to win, in 3 out of 4 cases, the individual FF factors and market portfolio during the IT bubble period. The claim of existing cointegration in the efficient markets is unclear. It has been presented different views for and against cointegration in efficient markets. For example (Granger 1986) and (Baillie, Bollerslev 1989) have argued that predictability of cointegration would mean inefficient markets. On the other hand, (Dwyer, Wallace 1992) and (Ferre, hall 2002) have argued that inefficiency and cointegration is not the same thing. In our empirical research, it seems to be historically unique, that the factors had very large deviations at the time of the IT bubble. Those previous features might refer to a market inefficiency. Also opinions on small cap value premium have been dividing the scientist opinions. For example, it is argued whether the value premium is from an anomaly or something other rise (such as an extreme loss at very bad times). If this is a pure anomaly, it should disappear. For example Table 1. shows clearly a restructuring to the small cap growth factor after the 80s, after which the shares of this factor have given a lot weaker returns than the other factors.

4. Conclusion

In this study we evaluate cointegration between shares prices of the Fama-French factors (Fama, French 1993, 1996) and the market in the period 1980-2004. The FF factors were used by the Russell style investment indices. Cointegration is especially useful for the review of non-stationary data, which the style investment indices are. The previous analysis will be a good addition, when one wanted to go further than the correlation analysis and possibly create some kind of model for predictions.

Initially the FF factors' historical returns were compared to the corresponding style indices returns. The results were otherwise similar except for the small cap growth portfolio, which gave given lower returns in our study, so that the small cap growth portfolio premium was significantly decreased since the 80s.

The main interest centered on cointegration between multiple time series using Johansen procedure (Johansen 1988; Johansen, Juselius 1990). First an error analysis was carried out, which was found that the errors were somewhat correlated. On the other hand neither autocorrelation nor normality was seen at sufficiently long lags. For the five time-series studied the cointegration was rank 2, which also means that there are two cointegration vectors.

Finally an error correction model (ECM) was created between the market portfolio and the indices, which were successfully used to create the style investing based hedge portfolios, which were used successfully within four different long training periods from three months time clips. In particular hedge strategies were quite successful, as during the IT bubble they beat in all cases the market portfolio and in three cases of four the best succeeded index. Consequently it was found that the components can form a stationary entirety and their behaviour is interdependent on each other.

Moreover all individual indices and market portfolio was tested between the two combinations of cointegrations, which didn't occur except one exception. Thus the ECM model cannot be created this situation.

Criticism, however, has been heard on the existence of cointegration in effective markets (Granger 1986; Baillie, Bollerslev 1989). But some researchers do not see a contradiction between cointegration and efficient markets (Dwyer, Wallace, 1992; Ferrer, hall 2002). Also small cap value share future premium has raised discussion, as it should disappear if it is an anomaly. For instance the small cap growth premium has radically diminished after the 80s.

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Appendix 1. Portfolios' weights in every quarter.

	training period															
	5 year				7 year				10 year				all data			
	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D
1991Q1				1				1				1				1
1991Q2		1			1				1				1			
1991Q3	1				1				1				1			
1991Q4			1		1					1						1
1992Q1	1				1					1						1
1992Q2		1				1				1						1
1992Q3		1				1			1				1			
1992Q4			1				1		1					1		
1993Q1			1				1		1						1	
1993Q2		1				1					1				1	
1993Q3			1				1		1						1	
1993Q4				1		1				1				1		
1994Q1			1			1				1				1		
1994Q2	1				1				1				1			
1994Q3				1		1					1				1	
1994Q4		1						1	1					1		
1995Q1				1		1			1						1	
1995Q2		1				1				1				1		
1995Q3				1		1				1				1		
1995Q4	1				1					1				1		
1996Q1				1				1		1			1			
1996Q2	1					1				1				1		
1996Q3	1				1					1			1			
1996Q4	1						1			1			1			
1997Q1				1		1				1				1		
1997Q2			1		1				1					1		
1997Q3	1				1				1				1			
1997Q4	1				1				1						1	
1998Q1				1	1							1			1	
1998Q2			1		1				1				1			
1998Q3		1				1				1				1		
1998Q4				1				1		1				1		
1999Q1		1						1		1						1
1999Q2		1				1				1				1		
1999Q3		1				1				1					1	
1999Q4	1				1				1				1			
2000Q1		1										1				1
2000Q2			1					1			1				1	
2000Q3	1						1		1						1	
2000Q4		1			1						1				1	
2001Q1			1				1				1				1	
2001Q2			1				1				1				1	
2001Q3		1					1					1			1	
2001Q4				1		1					1				1	

2002Q1		1					1					1			1	
2002Q2				1				1	1						1	
2002Q3			1		1						1				1	
2002Q4		1					1					1				1
2003Q1		1						1			1					1
2003Q2				1			1			1						1
2003Q3			1					1			1				1	
2003Q4		1					1			1					1	
2004Q1				1			1		1				1			
2004Q2	1					1			1				1			
2004Q3			1					1		1				1		

A = large cap value B = large cap growth
C = small cap value D = small cap growth

Table A1. Portfolios' weights 1991Q1-2004Q3.