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ABSTRACT

We study auction design when parties cannot commit to the mechanism. The seller may change the rules of the game any number of times and the buyers may choose their outside option at any stage of the game. A dynamic consistency condition and an optimality condition property are defined to characterize the seller's mechanism selection behavior. The unique stationary mechanism selection rule that meets the conditions is the English auction.

JEL: C72, D44, D78

Keywords: auctions, commitment, consistency, one-deviation property, stationarity

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1 Introduction

Trust and the ability to make credible promises is central to well functioning markets. However, these preconditions cannot always be fulfilled. Badly functioning enforcement system is a characteristic feature of many developing societies as well as ones undergoing an institutional change. It is also typical for anonymous platforms like the Internet.

By evidence and casual observations, the *ascending English auction* stands out as a particularly useful mode of trading across different cultures and development stages.[1] Ascending auctions were in widespread use in the days of the Roman Empire (the Latin root of the term "auction" means increasing). Yang (1950) describes the ascending auctioning of clothes in Chinese monasteries in the 600 AC. Today the English auction is the far most frequently used auction format in the Internet. For example, Lucking-Riley (2000) reports that almost 90% of the Internet auction sites used the English auction as their selling mechanism.

It is not clear why the English auction is such a dominant auction form, and why it is particularly so in circumstances of inadequate enforcement system. Popularity of the English auction is interesting since it is not, in general, the most profitable auction mechanism (Myerson, 1981).[2]

This paper develops a theory that attempts to explain this empirical regularity. The argument is not based on the profitability of the English auction but rather its *commitment* properties. We claim that the English auction is the *only* implementable mechanism if the market sides do not have any commitment power. Lack of commitment power could be due to an inadequate enforcement system.

Full commitment is a standard assumption in the mechanism design theory. In the auction set up it requires, on the one hand, that the seller (she) cannot change the rules of the game in the middle of the play and, on the other, that the buyers cannot leave the game once they participate it. By the commitment assumption, it is legitimate to abstract from issues of bargaining under incomplete information. However, it is not difficult to imagine that the seller, who has the power to design the rules of the mechanism ex ante, may be able to *re*design them too ex post, once the information has been processed but the *quo* has not been fulfilled. Familiar examples of auction manipulation include ex post bargaining over the good, shill bidding, fees, reauctioning the good, etc..[3] Redesign should be especially difficult to prevent under inefficient occurrences.

To see what goes wrong in the absence of commitment, recall that incentive compatibility and individual rationality are necessary for a (direct) mechanism to work. If the seller holds the power to redesign the rules of the mechanism, she wants to do so at least after any realization that leaves the buyers with positive surplus. Since forward looking buyers anticipate that the seller will redesign the

[1] Cassady (1967) is a standard reference on the history of auctions.

[2] However, the English auction may be optimal in a restricted class of mechanisms, see e.g. Milgrom and Weber (1982) or Lopomo (1998, 2001).

[3] See McAdams and Schwarz (2006) for analysis and discussion on changing the auction rules e.g. of the European 3G auctions. For another angle to shill bidding, see Izmalkov (2005).

mechanism ex post, they will adjust their play accordingly at the interim stage.[4] Consequently, incentive compatibility of the mechanism breaks down.[5]

We analyze auction design under the benchmark hypothesis that parties do not have *any* commitment power: the seller may freely change the auction rules as many times as she wishes, and the buyers can leave the auction at any point of the game. A novel framework is developed to analyze the problem. The idea is to first apply the standard revelation argument to isolate possible solutions to the seller's mechanism selection problem, and then restrict the solutions further with the consistency and optimization conditions that reflect the seller's dynamic behavior.

More precisely, we decompose the mechanism into an *information processing* device and an *implementation* device. The information processing device can be interpreted as a machine or a mediator that transforms the messages of the buyers into an output - a public signal - in a reliable and secure way. No technological constraints are imposed on the form of this information processing device. The task of the implementation device is to choose a physical outcome contingent on the signal that is generated by the information processing device. One should note that decomposability is not a restriction on feasible mechanisms: any mechanism can be decomposed in a unique way into the prescribed information processing and implementation devices. The crucial assumption - and the reason for decomposing the mechanism into two parts - is that the seller cannot commit to the implementation device. That is, *ex post* she can invoke another (composite) mechanism rather than implement the outcome suggested by the implementation device.

The sellers mechanism design behavior is captured by a rule σ that identifies, for each seller's belief p, a mechanism $\sigma(p)$ that the seller implements under belief p. Two conditions are imposed on the rule σ that guarantee that a sequentially rational seller can commit to it. The first is that the rule has to be *internally consistent*: selecting a mechanism according to the rule should not be in conflict with obeying the rule at later stages of the game. The second condition is that the rule needs to be *optimal*: under any belief p the seller should not be able to profit by deviating from $\sigma(p)$ *within* the class of mechanisms that she can commit to, given that she obeys σ in the future. The latter property is dubbed as the *one-deviation property.* A stationarity condition, which requires that the rule is not conditioned on payoff irrelevant information, is also assumed.

Our main result is that the payoff and information structure of any feasible mechanism, *i.e.,* mechanism chosen by a stationary and consistent mechanism selection rule that also satisfies the one-deviation property, is the English auction (indexed by a tie-breaking rule). Conversely, the rule that always chooses the English auction is consistent, stationary, and meets the one-deviation property. Thus the English auction is (essentially) the *unique* mechanism that the seller can implement without commitment.[6] Our model may therefore explain why the Vickrey or other prominent auction forms are rare but the English auctions are

[4] In Freixas et al. (1985) this is called the *ratchet effect.*

[5] Unless all the surplus is extracted from the buyers á la Cremer and McLean (1988). But this requires the *buyers* to commit to participate ex post.

[6] "Essentially" means that the auction may also reveal some immaterial information.

common (see Rothkopf et al., 1990).

Our argument is closely related to the famous *Coase conjecture*, arguing that in the one buyer scenario the seller without commitment power is forced to sell the good with the price equal to the least possible valuation of the buyer (see Gul et al., 1986; Fudenberg et al., 1985, Ausubel et al., 2002). For the same reason the English auction is robust against commitment problems. The English auction reveals (i) the buyer with the highest valuation (the winner), and (ii) valuations of all but the winner. Since the winner is known to have the highest valuation once the output has materialized, the seller cannot commit to sell the good anyone but the winner. Hence, as under the Coase conjecture, the seller is forced to sell the good to the winner with his least possible valuation which is equal to the second highest valuation of the buyers, already revealed by the mechanism. In the one buyer case, this argument collapses back to the standard Coase conjecture.[7]

Literature on mechanism design without commitment The distinguishing feature of this paper in the literature on mechanism design without commitment is that it does not put any restrictions on mechanisms that are technically feasible for the designer neither at the ex ante nor at the ex post stage; the problem is genuinely that of commitment. For example, there are neither discounting nor other waiting costs.

On the modeling side, the novelty of the paper is that the commitment problem is studied via revelation games rather than as a bottom-up extensive form game. The reason why we focus on a reduced form expression of the true underlying game is foremost expositional. But we do so also because we are not interested in identifying all the equilibria of the game. All features of the model, *i.e.,* multitudity of players, unboundedness of redesign rounds, and richness of the action sets (= all mechanisms) hint that this set would not be small. Hence, we follow the standard mechanism design avenue by assuming (implicitly) that *truthfulness,* whenever appropriately defined, is a focal feature of an equilibrium. In our framework, this means that it is the truthful equilibrium of the reduced form of the continuation game that is relevant. Hence, as in the standard mechanism design literature, we implicitly assume that the seller can choose the continuation equilibrium of the game. The key problem of the seller is to choose the continuation equilibrium in a dynamically consistent way when information is being revealed along the play. This is the interpretation of our equilibrium concept.

It should be emphasized that the methods developed in this work do not attempt to challenge or provide an alternative to the standard equilibrium techniques. Rather, the modeling here is meant to be consistent with them. The only reason for focusing on the reduced form expressions is to penetrate into the core aspects of the problem.

Of course, useful results can also be obtained by appealing to standard equilibrium techniques. However, this requires somewhat more restricted domain. McAfee and Vincent (1997) study an auction designer who can set a positive re-

[7] For studies on the no gap -case in the durable good monopoly scenario, see Ausubel and Deneckere (1989a,b)

serve price but cannot commit to not re-auction the good in the future when no current bids exceed the reserve price. Assuming a fixed auction mechanism, they demonstrate that as the lag before potential re-auction becomes short, the sequentially optimal (given re-auctioning) reserve-price produces the same expected revenue as an auction with a reserve price equal to the seller's valuation of the good.

McAdams and Schwarz (2006) study the first-price auction with a seller tempted to take further rounds of bids. The rational buyers will then prefer to wait before making their best and final offers which induces the seller to bargain at length with buyers. When the seller's cost of soliciting another round of offers is very small, the resulting equilibrium resembles that of the English auction.

Bester and Strausz (2001) and Skreta (2006) take a different route by allowing only two rounds. However, now the second stage mechanism need not be fixed. Bester and Strausz show that in the one buyer case the best mechanism is still direct (however, Bester and Strausz, 2000, show that with more than one agent, this no longer holds). But as opposed to the revelation principle, fully revealing contracts need no longer be optimal. In Skreta's framework, there is the additional problem of informed principal at the second stage. Skreta (2006) shows that the McAfee and Vincent auction is the optimal two-period mechanism for selling one unit of a good.

Skreta (2007) is the only paper we are aware of that develops techniques of analyzing multi-stage, multi-agent mechanism design problems without commitment. In her framework, the seller can re-auction the good if it has not been previously sold. The innovation is that, unlike in McAfee and Vincent (1997), the seller may employ any mechanism in way of doing this. A key assumption of Skreta's analysis is that there is an upper bound on the number of redesign rounds. For the sake of a tradeoff, there is also discounting and this transforms the seller's problem into one of intertemporal optimization. However, the central feature of our framework is that there is no bound on how many times the mechanism can be redesigned, and there is no cost of waiting.[8]

The problem of redesign is also akin to the literature on resale in auctions (Haile, 2000, and Zheng, 2002, are seminal contributions). Much of the focus in this literature has been in identifying the optimal auction with resale.[9] However, this literature is fundamentally different in one respect: once the good is sold to the buyers, the seller becomes privately informed which in general prevents efficiency (due to Myerson and Satterthwaite, 1983). Thus the problem no longer has the recursive structure that drives the analysis of this paper; that the design problem and *re*design problem are conceptually similar, and they should be solved by using common principles.

Further connections to the literature are discussed in the final section.

The paper is organized as follows: Section 2 specifies the set-up and the game. Section 3 defines the solution. The results are stated in Section 4. Section 5

[8] Unlike this paper, Skreta (2007) also allows the seller to become privately informed along the play, which is a considerable complication.

[9] See especially Zheng (2002) but also Calzolari and Pavan (2006), Garratt and Troger (2006), Pagnozzi (2007), Hafalir and Krishna (2008).

concludes with discussion.

2 Set up

There is a seller of a single indivisible good and a set $N = \{1, ..., n\}$ of buyers. Seller's publicly known valuation of the good is 0. Buyer i's privately known valuation θ_i is drawn from a discrete set $\Theta_i \subseteq \mathbb{R}_+$.[10] Write $\Theta = \times_{i \in N} \Theta_i$ with a typical element $\theta = (\theta_i)_{i \in N}$, and $\Theta_{-i} = \times_{j \neq i} \Theta_i$ with a typical element $\theta_{-i} = (\theta_j)_{j \neq i}$. [11] Denote by $\Delta(\Theta)$ the set of probability distributions p over Θ, and by p_i the ith marginal distribution of p.

The set of allocations of the good is $A = \{(a_1, ..., a_n) \in \{0,1\}^n : a_1 + ... + a_n \leq 1\}$, where $a_i = 1$ if the good is allocated to i and $a_i = 0$ otherwise. Write $a = (a_1, ..., a_n)$. A money transfer from buyer i to the seller is denoted by $m_i \in \mathbb{R}_+$ and $m = (m_1, ..., m_n)$ is a profile of transfers. The set of all outcomes $x = (a, m)$ is then $X = A \times \mathbb{R}^n_+$.

Now we define a *mechanism.* A mechanism does two things: processes information and implements an outcome. We separate these tasks. A mechanism is a composite function $\phi = g \circ h$, consisting of an information processing device h and an implementation device g such that

$$h : \Theta \to \Delta(S) \text{ and } g : S \to X,$$

where $\Delta(S)$ is the set of probability distributions over S, an open subset of an Euclidean space. That is, the information processing device h generates, after receiving the buyers' messages, a public signal $s \in S$. The signal s is the only information anyone - including the seller - obtains from h. The outcome function g then implements an outcome $x \in X$ conditional on the realized signal s.

Thus the mechanism $\phi = g \circ h$ is a composite function

$$g \circ h : \Theta \to \Delta(X),$$

where $\Delta(X)$ is the set of probability distributions over X. Letting $H = \{h : \Theta \to \Delta(S)\}$ and $G = \{g : S \to X\}$ denote the sets of information processing devices and implementation devices, respectively, the set of all composite mechanisms is

$$\Phi = \{g \circ h : \Theta \to \Delta(X) \text{ such that } g \in G \text{ and } h \in H\}.$$

The support of distribution p is denoted by supp(p). Also write $h(\theta) = \{s : h(s : \theta) > 0\}$ and $h(\text{supp}(p)) = \{s : h(s : \theta) > 0 \text{ and } \theta \in \text{supp}(p)\}$. Given p, a signal $s \in h(\text{supp}(p))$ of the information processing device h induces a posterior

$$p(\theta : s, h) = \frac{p(\theta) h\,(s : \theta)}{\sum_{\theta \in \Theta} p(\theta) h\,(s : \theta)}. \tag{1}$$

[10] Hence countable and without accumulation points. This assumption is for expositional simplicity.

[11] That is, $p_i(\theta_i) = \sum_{\theta_{-i}} p(\theta_i, \theta_{-i})$.

To economize on notation, write $p(\cdot : s, h) = p(s, h)$. Since the signals are public, $p(s, h) \in \Delta(\Theta)$ for all $s \in h(\text{supp}(p))$. By the definition of the support, $\text{supp}(p(s, h)) \subseteq \text{supp}(p)$.

The mechanism $g \circ h$ is *constant under* p if $h(\text{supp}(p))$ is singleton. A constant mechanism implementing outcome x is denoted by

$$\mathbf{1}_x \in \Phi.$$

A constant mechanism does not affect the beliefs and always implements the same outcome. The two mechanisms $(g \circ h)$ and $(g' \circ h')$ are *outcome equivalent* under p if they induce the same outcome function: $(g \circ h)(\theta) = (g' \circ h')(\theta)$, for all $\theta \in \text{supp}(p)$. Finally, if the information provided by the mechanism $g \circ h$ is not finer than what is necessary to implement the outcome, i.e., if $g(s) = g(s')$ implies $s = s'$ for all $s, s' \in h(\text{supp}(p))$, then we may write $p(s, h) = p(g(s), g \circ h)$.

Buyer θ_i's and the seller's payoffs from allocation $x = (a, m)$ are, respectively,

$$\begin{aligned} u_i(x, \theta_i) &= \theta_i a_i - m_i, \\ v(x) &= \sum_{i \in N} m_i. \end{aligned}$$

3 Solution

The seller's problem is that she cannot commit to the implementation device g once the signal s has been produced by the information processing device h. Rather, she may want to design a new mechanism under her post-signal belief. In this section, we identify conditions that the mechanism needs to satisfy if the seller is about to commit to it.

We appeal to the *revelation principle* by assuming that the mechanism is played truthfully if and only if (i) the appropriate incentive and participation constraints of the buyers are satisfied, (ii) the seller can commit to the mechanism. The latter requires that the seller *cannot* commit to implementing any more profitable mechanism given the post-signal beliefs. This means that the potentially more profitable mechanisms meeting (i) need to be blocked by a yet third layer of mechanisms which, in turn, the seller can commit to. The self-referential nature of the blocking-relationship between the mechanisms means that there is no recursive way to identify the feasible mechanisms. Indeed, the mechanism selection needs to be solved for all cases simultaneously. Thus to solve (ii), novel modeling techniques needs to be developed.

Buyers' incentives We assume that the buyers can exit any point of the game. Thus any implementable mechanism $g \circ h$ must be *ex post individually rational* (EXP-IR):[12]

$$u_i(g(s), \theta_i) \geq 0, \quad \text{for all } s \in h(\theta), \text{ for all } \theta \in \text{supp}(p), \text{ for all } i \in N.$$

[12] *Interim* individual rationality requires that participation be weakly profitable before the output has been realized. Ex post constraint has been analysed e.g. by Forges (1993, 1998) and Gresik (1991, 1996).

Given p, buyer θ_i's *interim* payoff from a mechanism $g \circ h$ is

$$\sum_{\theta} \sum_{s} p(\theta)\, u_i(g(s), \theta_i) h(s : \theta).$$

By the *revelation principle* (cf. Myerson, 1979), an implementable mechanism must be *incentive compatible.* A mechanism $g \circ h$ is incentive compatible (IC) if, for all $\theta_i, \theta_i' \in \Theta_i$, for all $i \in N$,

$$\sum_{\theta_{-i}} \sum_{s} p(\theta)\, u_i(g(s), \theta_i) \left[h(s : \theta) - h(s : \theta_{-i}, \theta_i')\right] \geq 0.$$

However, incentive compatibility and ex post individual rationality are not independent conditions: veto right might be exercised at the off-equilibrium nodes. The following simple extension of incentive compatibility resolves the problem by allowing i veto the outcome also after his untruthful announcements.[13]

Definition 1 (VETO-IC) *Given p, a mechanism $g \circ h \in \Phi$ is* veto-incentive compatible *if, for all $\theta_i, \theta_i' \in \Theta_i$, for all $i \in N$,*

$$\sum_{\theta_{-i}} \sum_{s} p(\theta) \left[u_i(g(s), \theta_i) h(s : \theta) - \max\{u_i(g(s), \theta_i), 0\} h(s : \theta_{-i}, \theta_i')\right] \geq 0. \qquad (2)$$

Veto-incentive compatibility requires that truthful reporting forms a Bayes-Nash equilibrium even if vetoing is possible after untruthful announcement. Any implementable mechanism must thus be veto-incentive compatible. For any p, denote the set of veto-incentive compatible mechanisms by

$$VIC[p] \subset \Phi.$$

It is easy to see that any veto-incentive compatible mechanism is incentive compatible and ex post individually rational (but not vice versa).[14]

Truthful announcements form a Bayes-Nash equilibrium in a veto-incentive compatible mechanism $\phi = g \circ h$ *if* the seller can *commit* to following g after h has performed its information processing task, i.e., produced its signal s. Thus a mechanism maximizing the seller's payoff subject to the veto-incentive compatibility could be interpreted as the seller's full commitment benchmark. Since veto-incentive compatibility concerns only the payoffs, any signal structure - even a one that fully reveals the buyers' types - is consistent with veto-incentive compatibility. However, while signals do not affect anyone's payoff directly, they may do so indirectly, via seller's behavior at the ex post stage.

Seller's incentives The seller's expected payoff from a mechanism $\phi = g \circ h$ is

$$v(\phi, p) = \sum_{\theta} \sum_{s} p(\theta) v(g(s)) h(g(s) : \theta).$$

[13] Veto-incentive compatibility is due to Forges (1998), and is closely related to IC* of Matthews and Postlewaite (1989).

[14] Choose $\theta_i = \theta_i'$ in (2). We only need EXP-IR and IC in the remainder of the paper.

She wants to maximize her expected payoff subject to the constraint of not redesigning the mechanism after observing the signal s from the information processing device h. That is, of replacing the outcome $g(s)$ with *another* mechanism in Φ that generates her a higher expected payoff than $g(s)$. Our task is to identify conditions under which she will not do that.

Let the seller's (pure) mechanism design strategy be captured by a *choice rule* σ that specifies, for each prior belief p, a mechanism that the seller *can* commit to under these beliefs:

$$\sigma : \Delta(\Theta) \to \Phi \quad \text{such that} \quad \sigma[p] \in VIC[p], \quad \text{for all } p. \tag{3}$$

Then $\sigma[p]$ represents the mechanism that the seller implements under p. Rule σ represent in reduced form the dynamic mechanism selection strategy of the seller.

We now identify properties that the choice rule σ should satisfy. We argue that sequential rationality of the seller, and the buyers' knowledge of this, asks σ to reflect internal consistency and maximization. To present these conditions, we develop some concepts. We say that a mechanism $g \circ h \in \Phi$ is (weakly) *ex post dominated* by a mechanism $\phi \in \Phi$ if there is a signal $s \in h(\text{supp}(p))$ such that

$$v(\phi, p(s,h)) \geq v(g(s)) \text{ and } \phi \neq \mathbf{1}_{g(s)} \text{ under } p(s,h).$$

That is, the seller weakly prefers ϕ over the recommended outcome $g(s)$, given the ex post information due to signal s. In such case, the original mechanism $g \circ h$ may be subject to *redesign*. It is easy to see that in a typical scenario, there is *no* veto-incentive compatible mechanism that is *not* ex post dominated.[15] Thus the seller is typically (weakly) tempted to redesign the mechanism.

Under prior p, denote by $C^\sigma[p]$ the set of mechanisms that are *not* subject to redesign under the hypothesis that σ is followed ex post:

$$C^\sigma[p] = \{g \circ h \in VIC[p] : g \circ h \text{ is not ex post dominated by } \sigma[p(s,h)], \text{ for any } s \in h(\text{supp}(p))\}. \tag{4}$$

Hence, by the revelation principle, and under the hypothesis that the seller can commit to the choice rule σ :

- a mechanism ϕ is truthfully playable if $\phi \in C^\sigma[p]$, since then it will not be redesigned ex post, and
- a mechanism ϕ is *not* truthfully playable if $\phi \notin C^\sigma[p]$, since then it will be redesigned ex post.

We now specify formally conditions that sequential rationality imposes on the choice rule σ. The first condition requires consistency in the sense that employing σ *ex ante* should not contradict σ being employed *ex post.*

Definition 2 (Consistency) *Choice rule σ is* consistent *if $\sigma[p] \in C^\sigma[p]$, for all p.*

[15] If $0 \in \text{supp}(p_i)$ for all i, then a veto-incentive compatible mechanism is not ex post dominated only if extracts all surplus from the buyers. But full surplus extraction á la Cremes and McLean (1984) is not possible under veto-incentive compatibility.

The second condition implies optimality. Given σ and p, the seller should choose a mechanism that maximizes her payoff in the set $C^{\sigma}[p]$.

Definition 3 (One-Deviation Property) *Choice rule σ satisfies the* one-deviation property *if $v(\sigma[p], p) \geq v(\phi, p)$, for all $\phi \in C^{\sigma}[p]$, for all p.*

Under the hypothesis that σ can be committed to in the future, the seller does not want to change σ under any current prior p. Without the one-deviation property σ could not be convincingly committed to. One-deviation property is in line with standard equilibrium reasoning. Indeed it is often drawn as a consequence of it.

Now we state two straightforward but important implications of consistency and the one-deviation property. First, the seller always implements the outcome of a mechanism that she can commit to.

Lemma 1 *Let σ be consistent and satisfy the one-deviation property. Then $g \circ h \in C^{\sigma}[p]$ implies that $\sigma[p(s,h)] = \mathbf{1}_{g(s)}$, for all $s \in h(\mathrm{supp}(p))$.*

Proof. Take any $s \in h(\mathrm{supp}(p))$. By consistency, $g \circ h$ is not ex post dominated by $\sigma[p(s,h)]$ under p. By the definition of ex post dominance, $\mathbf{1}_{g(s)}$ is not ex post dominated by $\sigma[p(s,h)]$. Hence either $v(g(s)) > v(\sigma[p(s,h)], p(s,h))$ or $v(g(s)) = v(\sigma[p(s,h)], p(s,h))$ and $\sigma[p(s,h)] = \mathbf{1}_{g(s)}$. By the one-deviation property, $v(\sigma[p(s,h)], p(s,h)) \geq v(g(s))$. Hence it must be the case that $\sigma[p(s,h)] = \mathbf{1}_{g(s)}$. ∎

In particular, the choice rule σ is *idempotent* in the following sense: if $\sigma[p] = g \circ h$, then $\sigma[p(s,h)] = \mathbf{1}_{g(s)}$, for all $s \in h(\mathrm{supp}(p))$. That is, running σ twice in a row rather than once will not affect the outcome.

Second, if the seller can commit to implementing an outcome, then that outcome must maximize her payoff in the class of individually rational outcomes.

Lemma 2 *Let σ satisfy the one-deviation property. Then $\sigma[p] = \mathbf{1}_x$ implies that $v(x) \geq v(y)$, for all $\mathbf{1}_y \in VIC[p]$.*

Proof. Let $\sigma[p] = \mathbf{1}_x$, $v(x) < v(y)$, and $\mathbf{1}_y \in VIC[p]$. Since $\sigma[p] \in VIC[p]$, and since neither $\mathbf{1}_x$ nor $\mathbf{1}_y$ are ex post dominated by $\mathbf{1}_x$, we have $\mathbf{1}_x, \mathbf{1}_y \in C^{\sigma}[p]$. But this violates the one-deviation property. ∎

We now check that our solution is consistent with the standard bargaining theory.

The Coase conjecture The Coase conjecture, which pertains to our $n = 1$ case, argues that when the seller is unable to commit not to sell the good, the buyer is able to extract all the surplus. That is, the outcome of the one-sided bargaining game is to sell the good with price $\underline{\theta}(p)$, the minimal possible valuation θ in the support of p. The Coase conjecture has been extensively studied in the non-cooperative bargaining literature, and verified in the so called "gap" case $\underline{\theta}(p) > 0$ e.g. by Fudenberg et al. (1985) and Gul et al. (1986).

The next proposition shows that the result can be derived also in our set up, without going into the details of the bargaining process. Thus consistency and the one-deviation property do capture the key aspects of sequential rationality.

Remark 1 (Gap-case) *Let $n = 1$. Let σ be a consistent choice function meeting the one-deviation restriction. Then $\sigma[p] = \mathbf{1}_{(1,\underline{\theta}(p))}$, for all p such that $\underline{\theta}(p) > 0$.*

That is, any $\sigma[p]$ sells the good to the buyer with the price equal to his minimal possible valuation. To see this, note that by Lemma 1, $\sigma[p(s,h)] = \mathbf{1}_{g(s)}$, for all $s \in h(\text{supp}(p))$. By Lemma 2, $g(s)$ maximizes v in the class of constant, individually rational mechanisms under $p(s,h)$. Since $\underline{\theta}(p(s,h)) > 0$ we have $g(s) = (1, \underline{\theta}(p(s,h)))$. But by imitating $\theta = \underline{\theta}(p) > 0$, any $\theta' \in$supp(p) can guarantee to be able to buy the good with price $\underline{\theta}(p)$. Hence by incentive compatibility, $g(h(\theta)) = (1, \underline{\theta}(p))$, for all $\theta \in$supp(p).

However, it is also well known that in the "no gap" -case, $\underline{\theta}(p) = 0$, other more complex equilibria can be constructed (see e.g. Ausubel and Deneckere, 1989). To avoid them, the literature often focusses on simple "stationary" equilibria (see e.g. Ausubel et al., 2001).

The problem with multiplicity of complex solutions applies also in our case when $\underline{\theta}(p) = 0$. It can be shown that for any $\lambda \in \Theta$ there is a choice rule σ^λ that is consistent and meets the one-deviation property, and sells to types $\theta \geq \lambda$ and never to types $\theta < \lambda$ of the buyer given the prior p (see Appendix B for precise exposition). However, all constructed σ^λ are complex, and require the seller to condition σ^λ on seemingly superficial information. To remove these complexities, our final restriction imposes a degree of simplicity on choice rules. It demands that the implemented outcome is not conditioned on information that does not provide more profitable transaction opportunities.

Definition 4 (Stationarity) *A choice rule σ is stationary if $\sigma[p] = \mathbf{1}_x$, $\sigma[p'] = \mathbf{1}_{x'}$, $v(x) \geq v(x')$, and* supp$(p') \subseteq$ supp(p) *imply $x = x'$.*

That is, signals that do not allow the seller to implement a more profitable choice do not affect the seller's choice. For example, the choice rule σ^λ above fails stationarity.[16] The next section characterizes the inducable stationary choice rules in the general $n \geq 1$ case.

4 Results

The English auction ϕ^E The *tie-breaking rule* w always selects one of the players with the highest valuation:

$$w(\theta) \in \arg\max_{j \in N \cup \{0\}} \theta_j, \text{ for all } \theta \in \Theta. \tag{5}$$

[16] For an analogous restriction, see Ausubel and Deneckere (1992).

Moreover, if $w(\theta) = 0$, then $\theta_j = 0$ for all $j \in N$.

Given a prior distribution p and a tie-breaking rule w, we now construct a deterministic mechanism $\phi^E(\cdot) = g^E\left(h^E(\cdot)\right)$, an *English auction*.[17] Label the elements of a subset of the signal set $S^E \subset S$ by

$$S^E = \{(w(\theta), \theta_{-w(\theta)}) : \theta \in \Theta\},$$

and define a deterministic information processing device $h^E : \Theta \to S^E$ such that

$$h^E(\theta) = (w(\theta), \theta_{-w(\theta)}), \text{ for all } \theta \in \Theta.$$

In order to specify the implementation device g^E, construct the winner's money transfer rule as follows: for any $i \in N$,

$$m^E(i, \theta_{-i}, p) = \min\{\theta_i' : (\theta_i', \theta_{-i}) \in w^{-1}(i) \cap \operatorname{supp}(p)\}, \text{ if } \theta \in w^{-1}(i). \tag{6}$$

That is, $m^E(i, \theta_{-i}, p)$ is the smallest possible valuation of i given the information that (i) i is the winner, (ii) the other buyers' types are θ_{-i}. The implementation device $g^E : S^E \to X$ now satisfies, for each buyer $j \in N$, for all $(i, \theta_{-i}) \in S^E$,

$$g_j^E(i, \theta_{-i}) = \begin{cases} (1, m^E(i, \theta_{-i}, p)), & \text{if } j = i, \\ (0, 0), & \text{if } j \neq i. \end{cases} \tag{7}$$

That is, $\phi^E(\theta) = g^E(h^E(\theta))$ allocates the good to the winner $i = w(\theta)$ who pays a price equal to his least possible valuation (i) given the other players' types, (ii) the fact that he is the winner, and (iii) p.[18] The corresponding payoffs are

$$u_i\left(\phi^E(\theta), \theta_i\right) = \begin{cases} \theta_i - m^E(i, \theta_{-i}, p), & \text{if } \theta \in w^{-1}(i) \cap \operatorname{supp}(p), \\ 0, & \text{if } \theta \notin w^{-1}(i) \cap \operatorname{supp}(p). \end{cases}$$

By construction, ϕ^E is *efficient* and the price paid by the winner is less than or equal to his valuation, and at least as high as the other buyers' valuations.[19] Moreover, since the winner i becomes publicly known with the signal $s = (i, \theta_{-i})$, the posterior belief $p((i, \theta_{-i}), h^E)$ satisfies

$$\operatorname{supp}(p((i, \theta_{-i}), h^E)) \subseteq w^{-1}(i). \tag{8}$$

Since the mechanism is straightforward, *i.e.*, there is a bijection between the domain and range of g^E, we may denote the posterior $p(s, h^E)$ by $p(x, \phi^E)$.

Mechanism ϕ^E has the familiar pivotal structure: a buyer's payment - and hence his payoff - is independent of his announcement as long as he wins (or loses). The impact of lying on his payoff cannot be positive since it either induces the buyer to win when he would like to lose or to lose when he would like to win. Since truthtelling forms an equilibrium,

$$\phi^E \in VIC[p].$$

[17] We relax w from the description of the English auction ϕ^E for notational simplicity.

[18] Note that $\phi^E(p)$ reveals only the winner's identity and the other players' valuations. Hence it cannot be interpreted as the Vickrey (second-price) auction which asks *all* buyers to reveal their valuations.

[19] When the valuations are correlated, there may be a gap between this and the second highest valuation

Feasible choice rules To highlight the fact that in the above definition the English auction is conditioned on the prior p, denote it by $\phi^E[p]$. Now the function $\phi^E : \Delta(\Theta) \to \Phi$ can be taken as the *the English auction -choice rule.* Construct a correspondence $C^{\phi^E} \subset VIC$ such that

$$C^{\phi^E}[p] = \left\{\phi \in VIC[p] : \phi \text{ is not ex post dominated by } \phi^E[p'], \text{ for any } p' \in \Delta(\Theta)\right\}.$$

Note that the English auction -choice rule is idempotent: $\phi^E[p(x, \phi^E[p])] = \mathbf{1}_x$, for all $x \in \phi^E[p](\text{supp}(p))$, for all p. That is, after running the English auction, a new English auction does not change the outcome. This implies that the English auction $\phi^E[p]$ is *not* ex post dominated by $\phi^E[p(x, \phi^E[p])]$. Hence it follows that $\phi^E[p] \in C^{\phi^E}[p]$ for all p. More compactly:

Lemma 3 *$\phi^E[\cdot]$ is consistent.*

If $\phi^E[p]$ also maximizes v in $C^{\phi^E}[p]$ for all p, then ϕ^E satisfies the one-deviation property. We now show that this necessarily holds: *any* mechanism in $C^{\phi^E}[p]$ is outcome equivalent - and hence payoff equivalent - with the English auction $\phi^E[p]$.

Lemma 4 *$\phi \in C^{\phi^E}[p]$ only if ϕ is outcome equivalent to $\phi^E[p]$, for all p.*

Proof. Relegated to Appendix A. ■

That is, given p, the only veto-incentive compatible mechanisms that are not ex post dominated by any $\phi^E[p']$ are the mechanism $\phi^E[p]$ itself and its versions that may additionally reveal some non-relevant information concerning the winner's type. Thus ϕ^E has a "fixed point" property.

We next demonstrate that any stationary σ that is consistent and meets the one-deviation property induces C^σ that contains ϕ^E, specified for some tie-breaking rule w. Heuristically, if ϕ ex post dominates $\phi^E[p]$, then ϕ must change $\phi^E[p]$'s allocation. Since any outcome x of $\phi^E[p]$ reveals the winner and his least possible valuation given the other buyers' valuations, ϕ must threaten the winner to sell the good to the buyer with the second highest valuation to force the winner to pay a higher price. However, the threat is not credible since when the winner declines the offer, the seller sells, by stationarity, to the winner with his least possible valuation.

Lemma 5 *Let a stationary choice rule σ be consistent and satisfy the one-deviation property. Then there is a tie-breaking rule w such that $\phi^E[p] \in C^\sigma[p]$, for all p.*

Proof. Construct w as follows: For any $\theta \in \Theta$, denote by $\mathbf{1}_\theta$ the degenerate prior such that $\text{supp}(\mathbf{1}_\theta) = \{\theta\}$. Then there is an outcome x_θ such that $\sigma[\mathbf{1}_\theta] = \mathbf{1}_{x_\theta}$. Let $w(\theta) = i$ if x_θ allocates the good to i. By Lemma 2, such $w(\theta)$ satisfies (5). Use this w to construct ϕ^E.

Suppose, to the contrary of the claim, that there is p such that $\phi^E[p] \notin C^\sigma[p]$. Then $\phi^E[p]$ is ex post dominated by $\sigma[p(x, \phi^E[p])]$ for some $x \in \phi^E[p](\text{supp}(p))$. Denote $\sigma[p(x, \phi^E[p])] = g \circ h$.

Let x allocate the good to player i. By stationarity, since $g \circ h$ ex post dominates $\mathbf{1}_x$, $g \circ h$ cannot be a constant mechanism. By IC there are $\theta' \in$supp$(p(x, \phi^E[p]))$, $s \in h(\theta')$, and $j \neq i$ such that $\theta'_i = \theta'_j$, and such that $g(s)$ allocates the good to j. By Lemma 1, $\mathbf{1}_{g(s)} = \sigma[p(x, \phi^E[p])(s, h)]$. Since

$$\mathbf{1}_x \in VIC[p(x, \phi^E[p])] \subseteq VIC[p(x, \phi^E[p])(s, h))],$$

it follows by Lemma 2 that $v(g(s)) \geq v(x)$.

By (8), supp$(p(x, \phi^E[p])) \subseteq Y_i$ and, by the definition of support,

$$\text{supp}(p(x, \phi^E[p])(s, h)) \subseteq \text{supp}(p(x, \phi^E[p])).$$

Thus, by the construction of w, EXP-IR, and Lemma 1, $\sigma[\mathbf{1}_{\theta'}] = \mathbf{1}_x$. However, since $g(s) \in h(\theta')$, also supp$(\mathbf{1}_{\theta'}) \subseteq$supp$(p(x, \phi^E[p])(s, h))$. This implies, by stationarity, that $g(s) = x$, violating the assertion that x allocates the good to i and $g(s)$ to $j \neq i$. ■

Now we prove that if a stationary σ is consistent and meets the one-deviation property relative to C, then there is a tie-breaking rule w such that no element of $C[p]$ is ex post dominated by ϕ^E for any q.

Lemma 6 *Let a stationary choice rule σ satisfy consistency and the one-deviation property. Then $C^\sigma[p] \subseteq C^{\phi^E}[p]$, for all p, for some tie-breaking rule w.*

Proof. Let $g \circ h \in C^\sigma[p]$ and $s \in h(\text{supp}(p))$. Denote $x = g(s)$ and $q = p(s, h)$. By Lemma 1, $\sigma[q] = \mathbf{1}_x$. Identify w as in Lemma 5. It suffices for us to show that $\mathbf{1}_x$ is not ex post dominated by $\phi^E[q]$. Suppose, to the contrary, that it is. By Lemma 5, $\phi^E[q] \in C^\sigma[q]$. By the definition of one-deviation property, $v(\sigma[q], q) \geq v(\phi^E[q], q)$. Thus $v(x) \geq v(\phi^E[q], q)$.

Take any $y \in \phi^E[q](\text{supp}(q))$. Then supp$(q(y, \phi^E[q])) \subseteq$supp$(q)$ and, hence,

$$x \in \{x' : \mathbf{1}_{x'} \in VIC[q]\} \subseteq \{x' : \mathbf{1}_{x'} \in VIC[q(y, \phi^E[p])]\}.$$

Since, by Lemma 1, $\sigma[q(y, \phi^E[q])] = \mathbf{1}_y$ and $\mathbf{1}_x \in VIC[q(y, \phi^E[q])]$, it follows by Lemma 2 that $v(x) \leq v(y)$. Since y was arbitrary, and $v(x) \geq v(\phi^E[q], q)$, the inequality must hold as equality. But then, since supp$(q(y, \phi^E[q])) \subseteq$supp$(p)$, stationarity implies that $x = y$. Thus $\phi^E[q] = \mathbf{1}_x$, which contradicts the hypothesis that $\phi^E[q]$ ex post dominates $\mathbf{1}_x$. ■

By Lemma 6, a stationary and consistent σ that meets the one-deviation property is not ex post dominated by some English auction. Hence $\sigma[p]$ cannot allocate the good to anyone but the buyer with the highest valuation. Therefore, $\sigma[p]$ must be efficient. Another way to put this is that commitment inability of the seller leads to an efficient allocation, as suggested by the "Coase theorem".

Since $\sigma[p]$ is efficient, and the lowest type of a buyer earns zero payoff, the *revenue equivalence theorem* implies that $\phi^E[p]$ is the (generically) *unique* implementable mechanism if the buyers' valuations are independent. However, we can say more: by Lemma 4, if the seller is unable to commit, the uniqueness of the implementable mechanisms is a *general* phenomenon and holds for *any* prior distribution.

For an illustrative example, let $N = \{1, 2\}$ and $\Theta = \text{supp}(p) = \{5, 10\}^2$. Let, say, $w(10, 10) = w(10, 5) = w(5, 5) = 1$ and $w(5, 10) = 2$. Take $\phi \in C^{\sigma}[p]$. Since $\mathbf{1}_x$ is not ex post dominated by $\phi^E[p(x, \phi)]$ for any $x \in \phi(\text{supp}(p))$, $\phi(\theta)$ allocates the good to buyer 1 under all $\theta \in \{(5, 5), (10, 5), (10, 10)\}$. Transfers from 1 under $\theta = (5, 5)$ and $\theta = (10, 10)$ are 5 and 10, respectively. By incentive compatibility, transfer from 1 under $\phi(10, 5)$ is 5. Since only $\phi(5, 10)$ allocates the the good to 2, 2s type $\theta_2 = 10$ is then revealed. Hence his transfer must be 10 which means that $\phi = \phi^E[p]$ under p.

Now we are ready to state our main result.

Theorem 1 *1.Choice rule $\phi^E[\cdot]$ is consistent and satisfies the one-deviation property, for any tie-breaking rule w.*

2. If a stationary choice rule σ is consistent and satisfies the one-deviation property, then there is a tie-breaking rule w such that $\sigma[p]$ is outcome equivalent to $\phi^E[p]$, for all p.

Proof. 1. Consistency follows from Lemma 3. Since, by Lemma 4 any $\phi \in C^{\phi^E}[p]$ agrees with $\phi^E[p]$ on X and hence induces the same payoff as $\phi^E[p]$, it follows that $\phi^E[p]$ maximizes v on $C^{\phi^E}[p]$ under p. Thus the one-deviation property is implied.

2. Let stationary rule σ be consistent and satisfy the one-deviation property. By Lemma 5, there is w and C^{σ} such that $\phi^E \in C^{\sigma}$. By Lemma 6, $C^{\sigma} \subseteq C^{\phi^E}$. By construction, $\sigma \in C^{\sigma}$. Thus, by Lemma 4, $\sigma[p]$ is outcome equivalent to $\phi^E[p]$, for all p. ■

That is, the seller can commit to the English auction provided that she does that consistently, under all scenarios. Moreover, the payoff structure of every feasible auction coincides that of the English auction (defined for some tie breaking rule w). The only difference of a committable mechanism and the English auction may concern additional, payoff irrelevant information on the player's valuations.

It is interesting that while full surplus extraction is feasible under full commitment under almost all p (Crémer and McLean, 1988), only the English auction is feasible without commitment.[20]

The generalized Coase conjecture With the stationarity assumption the Coase conjecture can now also be verified in the "no gap" case. By Theorem 1.2., if σ is stationary, consistent, and meets the one-deviation property, then $\sigma[p] = \phi^E[p]$ which always allocates the good to the buyer $\theta > 0$ with price $\min w^{-1}(1) \cap \text{supp}(p)$.

To conclude, if one accepts the Coase conjecture, that the mechanism $\phi^E[p]$ is the unique feasible mechanism in the $n = 1$ case when the seller cannot commit to not sell to the buyer who values the good more than she does, then there is no reason not to accept also a more general version of the claim saying that in

[20] See also McAfee and Reny (1992).

the $n \geq 1$ case $\phi^E[p]$ is still the unique feasible mechanism when the seller cannot commit to not sell to the buyer who values the good more than the buyer with the second highest valuation. This suggests a generalization of the Coase conjecture: *Without external commitment devices, the seller can commit only to the English auction.*

5 Discussion

Mechanism design requires commitment since at the ex post stage, when the mechanism has produced information needed for choosing the output, the seller may want to change the rules of the game and implement a new mechanism. We have studied auction mechanisms that the seller can commit to implement. Two conditions reflecting sequential rationality of the seller have been imposed on the feasible mechanism selection rule. The conditions are internal consistency and an optimality condition called one-deviation property. We show that the unique mechanism that satisfies the restrictions (and a stationarity condition) is a version of the traditional English auction.

At the heart of the analysis is the argument that a sequentially rational seller can always commit to the English auction when her choices are stationary. This idea can be seen as a generalization of the Coase conjecture (e.g. Fudenberg et al., 1985; Gul et al., 1986). In the one buyer case, the seller cannot commit (under stationary strategies) to not to sell the good to the buyer with strictly *positive* valuation. In the multiple buyers case, the seller cannot commit not to sell the good to the buyer with the *highest* valuation. The reason is that the seller *can* always commit to the English auction and hence she *cannot* commit to mechanisms that are ex post dominated by the English auction. Our main result is that this constraint is very severe: only versions of the English auction satisfy it.[21]

One may wonder whether the ex post domination criterion in the definition of one-deviation property is needlessly strong. A natural weaker candidate would be to demand *strict* payoff dominance. Strict domination would, however, be in conflict with our basic assumption that the seller's mechanism selection rule is dependent only on the prior p. To see this, consider the $n = 2$ case and $\text{supp}(p) = \{0\} \times \{0, 1\}$. With the tie-breaking rule w that allocates the good under $\theta = (0, 0)$ to buyer 2, mechanism $\phi^E[p]$ would always sell to buyer 2 with price 0. With strict domination criterion, a procedure that sells to 1 under $\theta = (0, 0)$ with price 0 and to 2 under $\theta = (0, 1)$ with price 1 would be not be strictly ex post dominated. And selling to 1 under $\theta = (0, 0)$ with price 0 would be in conflict with $\phi^E[q]$ where prior q is degenerate on $\theta = (0, 0)$. Combining strict dominance with sequential rationality would therefore require history dependent choices, and the mechanism selection rule σ would no longer be definable as a function of p alone. However, while this seems to be technically burdensome, an analogue of Theorem 1 should hold with a history dependent tie-breaking rule. I conjecture that the English

[21] Milgrom (1987) argues that the core implements the efficient allocation under complete information.

auction with some tie-breaking would rule still be the unique feasible auction form.

Our model provides support to the English auction in the *general class* of auction mechanisms. Many studies have demonstrated the usefulness of the English auction in a restricted class of mechanisms. In a classic treatise, Milgrom and Weber (1982) show that the English auction is optimal among the four standard auction forms when the valuations are affiliated, a natural assumption in many auction scenarios.[22] In the same model, Lopomo (1998, 2001) demonstrates that the English auction features *robustness* in a sense that it is optimal among simple sequential auctions and in a class of *posteriorly implementable* auctions (a concept due to Green and Laffont, 1987).

Posterior implementability requires that the buyers' behavior is regret-free in a sense that they would not want to change their behavior even if they knew the outcome of the mechanism. This property is at the heart of the robustness of the English auction and the Vickrey auction.[23] It is partly driving also our results. Due to posterior implementability, running the English or the Vickrey auction twice in a row rather than just once will not affect the outcome. This implies that the English auction is *idempotent* in the sense of Lemma 1. But this is only a necessary condition of a feasible auction. On the sufficient side, one needs to guarantee that the seller cannot commit to any *more* profitable auction at the ex post stage, given the information that is generated by the mechanism. That the English auction does well also in this respect is not a consequence of posterior or ex post implementability. For example, the Vickrey auction reveals too much information as also the winner's type is revealed. Thus the unique feasibility of the English auction is due to both its posterior implementability and its innate informational properties.

But one can also see the situation the other way around. Since Wilson (1987), a recurrent theme in mechanism design literature has been that a good theory should not rely too heavily on assumptions of common knowledge. Motivated by the Wilson Doctrine, many authors have proceeded by imposing conditions such as ex post implementability on choice rules, as explained by Chung and Ely (2007), which presumably mirrors detail-freeness. It is, however, not completely clear what kind of *behavioral* implications does the restriction have. Is there a theoretical reason arising from the players' behavior that justifies detail-freeness as an assumption? Our model provides one such motivation: detail-freeness is bad if the seller cannot commit to the mechanism. In the absence of commitment, an implementable mechanism needs to have a degree of vagueness and hence only the ex post implementable English auction is feasible. Note that this applies also to the case of correlated valuations.[24]

[22] Including the English, Vickrey, Dutch, and first-price auctions. However, Matthews (1987) and Maskin and Riley (1984) show that *risk-aversion* reverses the ranking.

[23] The English auction also satisfies a stronger condition of *ex post implementability*: that one does not want to change his own behavior even if one knows the behavior of the other players. See e.g. Bergeman and Morris (2005, 2008).

[24] Chung and Ely (2007) provide a related motivation, stemming from the seller's uncertainty aversion.

Finally, our model provides also some insights into the literature on optimal auctions under efficiency (e.g. Ausubel and Cramton 1999; Krishna and Perry, 1998). The efficiency restriction is usually motivated vaguely by appealing to "Coasian dynamics", which leads to efficient allocation of resources through the seller's commitment inability, or resale markets.[25] This paper is explicit on how efficiency emerges as a consequence of sequentially rational redesigns of auction mechanisms.

[25]Zheng (2002) is an exception. He characterizes outcome functions that can be implemented with explicit resale markets. See also Haile (2000) for a formal modelling of retrading.

A Appendix

Proof of Lemma 4

First assume that $\phi = g \circ h$ is deterministic, i.e., $\phi(\theta) = g(h(\theta))$ is singleton for all θ. We show that $g(h(\theta)) = \phi^E[p](\theta)$, for all $\theta \in \Theta$. Denote, for notational simplicity, $Y_i = \{\theta \in \Theta : w(\theta) = i\}$ for all i.

Since $g \circ h$ is not ex post dominated by $\phi^E[p(s,h)]$ for any $s \in h(\mathrm{supp}(p))$, $g(h(\theta))$ has to allocate the good to the same buyer as $\phi^E[p]\,(\theta)$ does, for all θ. Thus the partition $\{Y_i\}$ specifies the winner under $g \circ h$. Since, by EXP-IR, a non-winner cannot be imposed a strictly positive monetary transfer, the allocation of $g(h(\theta))$ may differ from $\phi^E[p]\,(\theta)$ only in the winner's monetary transfer. Denote the winner's monetary transfer under $g(h(\theta))$ by $m_i(\theta)$. Our task reduces to showing that $m_i(\theta) = m^E(i, \theta_{-i}, p)$, for all $\theta \in Y_i$, for all i.

Fix i. Since Θ_i is discrete and bounded below, we can order its elements by $\theta_i^0 < ... < \theta_i^k < ...$. We prove by induction that $m_i(\theta_i^k, \theta_{-i}) = m^E(i, \theta_{-i}, p)$, for all θ_{-i} such that $(\theta_i^k, \theta_{-i}) \in Y_i$, for all $k = 0, 1, ...$. Assume that the induction hypothesis holds until step $k-1$, i.e.,

$$m_i(\theta_i^l, \theta_{-i}) = m^E(i, \theta_{-i}, p), \text{ for all } (\theta_i^l, \theta_{-i}) \in Y_i \cap \mathrm{supp}(p), \text{ for all } l = 0, ..., k-1. \tag{9}$$

We show that (9) holds also for step k.

Take any $s \in h(\mathrm{supp}(p))$. Since ϕ does not leave surplus to the winner that could be extracted by $\phi^E(p((x,s),\phi))$,

$$m_i(\theta_i^k, \theta_{-i}) \geq m^E(i, \theta_{-i}, p(s,h)), \text{ for all } (\theta_i^k, \theta_{-i}) \in \mathrm{supp}(p(s,h)).$$

Since $\mathrm{supp}(p(s,h)) \subseteq \mathrm{supp}(p)$,

$$m^E(i, \theta_{-i}, p(s,h)) \geq m^E(p, \theta_{-i}), \quad \text{for all } (\theta_i^k, \theta_{-i}) \in \mathrm{supp}(p(s,h)). \tag{10}$$

Noting that (10) holds for all $s \in h(\mathrm{supp}(p))$, it follows from the above two conditions that

$$m_i(\theta_i^k, \theta_{-i}) \geq m^E(i, \theta_{-i}, p), \text{ for all } (\theta_i^k, \theta_{-i}) \in \mathrm{supp}(p). \tag{11}$$

It remains to be shown that the weak inequality in (11) holds as equality.

By (6),

$$m^E(i, \theta_{-i}, p) = \theta_i^k \text{ for all } (\theta_i^k, \theta_{-i}) \in Y_i \cap \mathrm{supp}(p) \text{ such that } (\theta_i^{k-1}, \theta_{-i}) \notin Y_i. \tag{12}$$

This has two implications. First,

$$\sum_{(\theta_i^k, \theta_{-i},) \in Y_i} [\theta^k - m^E(i, \theta_{-i}, p)] p(\theta_i^k, \theta_{-i}) = \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i} [\theta^k - m^E(i, \theta_{-i}, p)] p(\theta_i^k, \theta_{-i}). \tag{13}$$

Second, by (11), $m_i(\theta_{-i}, \theta_i^k) \geq \theta_i^k$ for all $m_i(p, \theta_{-i}) = \theta_i^k$ for all $\left(\theta_i^k, \theta_{-i}\right) \in Y_i \cap \text{supp}(p)$ such that $\left(\theta_i^{k-1}, \theta_{-i}\right) \notin Y_i$. By VETO-IC and this property,

$$\begin{aligned} & \sum_{(\theta_i^k, \theta_{-i}) \in Y_i} [\theta_i^k - m_i(\theta_i^k, \theta_{-i})] p(\theta_i^k, \theta_{-i}) \\ \geq & \sum_{(\theta_i^k, \theta_{-i}) \in Y_i} \max\{\theta_i^k - m_i(\theta_i^{k-1}, \theta_{-i}), 0\} p(\theta_i^k, \theta_{-i}) \\ = & \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i} \max\{\theta_i^k - m_i(\theta_i^{k-1}, \theta_{-i}), 0\} p(\theta_i^k, \theta_{-i}). \end{aligned} \tag{14}$$

By the induction hypothesis (9),

$$\theta_i^k - m_i(\theta_i^{k-1}, \theta_{-i}) = \theta_i^k - m^E(i, \theta_{-i}, p), \text{ for all } (\theta_i^{k-1}, \theta_{-i}) \in Y_i \cap \text{supp}(p). \tag{15}$$

By (15) and (12),

$$\begin{aligned} & \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i \cap \text{supp}(p)} \max\{\theta_i^k - m_i(\theta_i^{k-1}, \theta_{-i}), 0\} p(\theta_i^k, \theta_{-i}) \\ = & \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i \cap \text{supp}(p)} [\theta_i^k - m^E(i, \theta_{-i}, p)] p(\theta_i^k, \theta_{-i}) \\ = & \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i} [\theta_i^k - m^E(i, \theta_{-i}, p)] p(\theta_i^k, \theta_{-i}). \end{aligned}$$

Thus,

$$\begin{aligned} & \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i} \max\{\theta_i^k - m_i(\theta_i^{k-1}, \theta_{-i}), 0\} p(\theta_i^k, \theta_{-i}) \\ \geq & \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i} [\theta_i^k - m^E(p, \theta_{-i})] p(\theta_i^k, \theta_{-i}). \end{aligned}$$

This together with (14) imply that

$$\sum_{(\theta_i^k, \theta_{-i}) \in Y_i} [\theta_i^k - m_i(\theta_i^k, \theta_{-i})] p(\theta_i^k, \theta_{-i}) \geq \sum_{(\theta_i^{k-1}, \theta_{-i}) \in Y_i} [\theta_i^k - m^E(i, \theta_{-i}, p)] p(\theta_i^k, \theta_{-i}).$$

Thus, by (13),

$$\sum_{(\theta_i^k, \theta_{-i}) \in Y_i} [\theta_i^k - m_i(\theta_i^k, \theta_{-i})] p(\theta_i^k, \theta_{-i}) \geq \sum_{(\theta_i^k, \theta_{-i}) \in Y_i} [\theta_i^k - m^E(i, \theta_{-i}, p)] p(\theta_i^k, \theta_{-i}),$$

and hence

$$\sum_{(\theta_i^k, \theta_{-i}) \in Y_i} m_i(\theta_i^k, \theta_{-i}) p(\theta_i^k, \theta_{-i}) \leq \sum_{(\theta_i^k, \theta_{-i}) \in Y_i} m^E(i, \theta_{-i}, p) p(\theta_i^k, \theta_{-i}). \tag{16}$$

Finally, (16) implies that (11) holds as equality, as desired.

Finally we check the case of random h. Note that even if $g \circ h$ is random, it has to allocate the good to the same buyer as $\phi^E[p]$ does. Thus randomness of $g \circ h$ may only concern the monetary transfer m. The proof, which is by induction, proceeds along the above lines. It is easy to verify that (11) holds also for any randomly generated monetary transfer m under $\left(\theta_i^k, \theta_{-i}\right)$. Moreover, (16) needs to hold for the *expected* transfer $\bar{m}$ under $\left(\theta_i^k, \theta_{-i}\right)$. Again, this just means that (11) holds as equality for each m, thus $m^E(i, \theta_{-i}, p)$ is implemented with probability one under all $\left(\theta_i^k, \theta_{-i}\right) \in$supp$(p)$. This completes the proof.

B Appendix

Non-stationarity of σ^λ

Let $n = 1$ and assume the "no gap" -case $0 \in \Theta \neq \{0\}$. We construct a seller's choice function σ^λ that allows the seller to commit to *any* price $\lambda \in \Theta$. Define a take-it-or-leave-it offer

$$\phi^\lambda(\theta) = \begin{cases} (1, \lambda), & \text{if } \theta \geq \lambda, \\ (0, 0), & \text{if } \theta < \lambda. \end{cases}$$

That is, "sell with price λ to any type θ at least λ and do not sell to anyone else". Define σ^λ such that

$$\sigma^\lambda[p] = \begin{cases} \phi^\lambda, & \text{if } 0, \lambda \in \text{supp}(p), \\ \mathbf{1}_{(1, \underline{\theta}(p))}, & \text{otherwise.} \end{cases}$$

We claim that σ^λ satisfies the one-deviation property.

(i) If $0, \lambda \in$supp(p), then $\{(1, \lambda), (0, 0)\} = \sigma^\lambda[p]$. Now:

- $0 < \lambda = \underline{\theta}(p((1, \lambda), \sigma^\lambda[p]))$ and thus $\sigma^\lambda[p((1, \lambda), \sigma^\lambda[p])] = \mathbf{1}_{(1,\lambda)}$, and

- $0 = \underline{\theta}(p((0, 0), \sigma^\lambda[p]))$ and thus $\sigma^\lambda[p((0, 0), \sigma^\lambda[p])] = \mathbf{1}_{(0,0)}$.

(ii) If $0 \notin$supp(p) and/or $\lambda \notin$supp(p), then $\sigma^\lambda(p) = \mathbf{1}_{(1,\underline{\theta}(p))}$ and $\sigma^\lambda(p) = \mathbf{1}_{(0,0)}$, respectively.

Since constant mechanisms do not affect beliefs, σ^λ satisfies the one-deviation property. Thus, the seller can commit to any price $\lambda \in$supp(p).

We now argue that σ^λ is not stationary. To see this, let $0, \lambda \in$supp(p). Find the degenerate prior $\mathbf{1}_0$ such that supp$(\mathbf{1}_0) = \{0\} \not\ni \lambda$. Then supp$(\mathbf{1}_0) \subset$supp$(p)$. However, $\sigma^\lambda[\mathbf{1}_0] = \mathbf{1}_{(1,0)}$ and $v((1, 0)) = v((0, 0)) = 0$, violating stationarity.

This result is analogous to Ausubel and Deneckere (1989), who show that any price can be supported in equilibrium in the one sided offers bargaining game when the discount factor δ tends to 1. Strategies needed for these equilibria are complicated, i.e. non-stationary.

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