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Aboa Centre for Economics

Discussion Paper No. 52
Turku 2009

ISSN 1796-3133

Printed in Uniprint
Turku 2009

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ABSTRACT

We consider how the inter-temporal discreteness of the revenue and cost processes affect the optimal timing of a real estate investment opportunity in comparison with the investment timing strategy obtained by relying on the traditional continuous real option model. We characterize both optimal investment rules explicitly and show that the continuous model may lead to a significantly higher required rate of return than the discrete model. Hence, our results show that the use of continuous time models leads to smaller and suboptimal amount of investment. Our numerical illustrations also indicate that this difference grows as volatility increases. Consequently, even though higher volatility decelerates investment in the discrete case as well, it decelerates it less than the continuous model would predict.

JEL Classification: G11, R31, C44.

Keywords: Real options, real estate investment timing, exchange option.

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Acknowledgements

Financial support from the Foundation for the Promotion of the Actuarial Profession for Luis H. R. Alvarez is gratefully acknowledged. All authors acknowledge the financial support from the OP Bank Group Research Foundation and the city of Helsinki.

1 Introduction

It is known that land can be more valuable as a potential site for development in the future than it is as an actual construction site at the present moment. Because of this, investors often choose to keep valuable land undeveloped for exceptionally prolonged periods emphasizing the real option value of waiting associated with the development of vacant land. In light of this observation, it is natural to interpret the optimal timing of construction as the optimal exercise strategy of a real option.

The real option nature of construction opportunities has not went unnoticed in the previous literature on real estate investment. The optimal timing of construction has been analyzed in terms of continuous time models in Capozza and Schwann (1989), Capozza and Li (2002), Grenadier (1996), and Williams (1993). In this literature it has been shown that uncertainty, for example regarding rental price growth or optimal building height, tends to delay the optimal timing of construction. The reason for this qualitative finding is that even though higher uncertainty may increase the expected cumulative returns resulting from the development decision, it simultaneously increases the real option value of land even in greater proportions thereby resulting in prolonged waiting. A second result of the previous literature emphasizing the role of strategic interaction is that competition among developers tends to accelerate the optimal timing of construction.

Titman (1985) provides an early application of option pricing approach to real estate development. The intuition behind Titman's simple two-date model is that a vacant lot can be viewed as an option to purchase one of a number of different possible buildings at exercise prices that equal their respective construction costs. Titman's paper shows that increased uncertainty leads to a decrease in building activity in the current period. Therefore, by decreasing uncertainty regarding the optimal height of buildings the initiation of height restrictions may lead to an increase in building activity in the area. Also Capozza and Li (2002) derive a model in which the uncertainty regarding optimal building size increases the real option value of vacant land. Based on the models by Capozza and Schwann (1989) and Capozza and Sick (1994), in turn, the uncertainty premium that increases the value of land and delays the optimal timing of construction increases with the variance of rental growth. Furthermore, in the Capozza and Helsley (1990) framework waiting may be optimal in the presence of uncertainty, since it reduces the probability of converting agricultural land to housing prematurely.

Williams (1993) derives an equilibrium set of exercise strategies for real estate developers. In Williams' model, all constructors build at the maximum feasible rate when income in the market rises above a critical value. The model also suggests that the critical value is different if the developers own developed assets.

In contrast with Williams' framework, in Grenadier's (1996) equilibrium model real estate development options may be exercised sequentially or simultaneously, depending on the underlying market conditions. Importantly, Grenadier further demonstrates that strategic interaction between (real estate) option holders moves optimal timing of construction earlier. In other words, Grenadier's model implies that competition between constructors diminishes the required return on new real estate development. The model also rationalizes the fact that sometimes real estate construction sits idle for years and then suddenly massive construction goes off sometimes even when the demand for space is falling in the market. Grenadier's model considers the option to replace an existing building by a new superior building that yields higher rental cash flows. Nevertheless, the same framework is also suitable in the case of an option to renovate an existing building and of an option to exercise new construction to an initially vacant lot. In the latter case the initial rent is simply set to zero.

Keuschnigg and Nielsen (1996), in turn, demonstrate that depreciation of housing structures weaker the incentives to keep land vacant and save it to future building purposes. That is, also depreciation lowers the required rate of return for housing development.

In addition, Capozza and Li (2002) show that, in the case of growing expected rental cash flows, development is optimally delayed beyond the point where net present value becomes nonnegative. This holds even under certainty. This is in line with the literature showing that the ability to delay an irreversible investment expenditure invalidates the usual net present value rule to invest when the net present value of an investment is nonnegative[1].

Only a limited number of papers study empirically the effect of real options on real estate investment. The empirical studies mainly concentrate on examining the effect of uncertainty on construction activity. The results by Holland et al (2000) and Cunningham (2006) are accordant

[1] An extensive review of this literature can be found in Dixit and Pindyck (1994) and Pindyck (1991) (see also Ingersoll and Ross (1992)).

with theory, implying that greater uncertainty reduces current development. Furthermore, Quigg (1993) estimates a real-option premium of six percent over the deterministic price for vacant land. Gunterman (1997), in turn, finds evidence for the hypothesis by Capozza and Helsley (1989) according to which land prices in rapidly growing areas include a significant premium based upon expectations of future growth. The estimations of Guntermann suggest that the premium varies from less than 40% of land value during busts to over 70% during booms in the case of Phoenix, Arizona.

All of the theoretical models (except for that by Titman (1985)) mentioned above assume the continuity of time. However, due to the special characters of the housing market, it would be more realistic to approach the optimal timing of real estate construction in a discrete time framework. More precisely, continuous time models are well suited for liquid markets, such as the stock market, where the price of the underlying asset is continuously settled as a result of frequent trading. In the relatively illiquid housing market, where the assets traded in the market are heterogeneous and continuously updated market information is lacking, typically only quarterly price information is available. In light of this observation, it is plausible to argue that continuous time models are to some extent unrealistic for modeling the optimal exercise of construction opportunities. Rather, due to the special characteristics of the housing market, the optimal timing of housing construction is better characterized by a discrete time model. This observation is naturally valid for most major discrete investment opportunities as well. Unfortunately, discrete time timing problems are typically technically more demanding than their continuous time counterparts. This, without doubt, is one of the main reasons for the fact that the previous literature has overlooked this important distinction and mainly focused on continuous time models. Our main objective is to address this challenging problem and rigorously analyze the quantitative significance of the difference between the optimal timing policies within these two different dynamical settings.

In our models both revenues as well as costs associated with the particular investment opportunity are allowed to follow two potentially correlated stochastic processes. In the discrete time setting these processes are modeled as two first order autoregressive processes driven by two correlated and normally distributed random walks. In line with the considered discrete time model, the underlying processes are then modeled as two geometric Brownian motions driven by a pair of correlated Wiener processes in the continuous time setting. The optimal timing of the investment opportunity is then modeled as the determination of the optimal exercise strategy of a generalized perpetual American exchange option written on two dividend paying assets. In the continuous time case the problem is reduced to the one originally considered in McDonald and Siegel (1986) (which has been later extended in Hu and Øksendal (1998) and Olsen and Stensland (1992)). However, to our best knowledge the discrete time problem considered in our study has not been previously analyzed nor solved within an optimal timing framework.

One of our main conclusions is that the use of a discrete time model leads to a lower required rate of return for the investment opportunity and thereby to earlier exercise than the conventional continuous time model would suggest. This result is of particular importance within a real estate investment framework, since it clearly indicates that the use of continuous time models leads to smaller and suboptimal amount of housing supply. The quantitative difference in the optimal timing policies between the discrete and continuous models depends on a number of exogenous variables: the volatilities of the revenues and costs associated to the particular investment opportunity, the prevailing riskless return, as well as the expected growth rate of revenues and costs. Our results indicate that this difference may be substantial and that it tends to increase as volatility increases. Consequently, the more volatile the markets are, the larger is the difference between the two optimal timing rules. This qualitative observation is important since it demonstrates that even though higher volatility decelerates investment in the discrete case as well, it decelerates it less than the continuous model would predict. The main reason for this result is the difference in the way the underlying revenue-cost-ratio dynamics approaches and finally exceeds a given fixed investment threshold characterizing the investment rule. In the continuous model a given investment threshold is approached continuously and, therefore, the actual timing rule can be interpreted as the first date at which the revenue-cost-ratio hits a predetermined optimal threshold level. In the discrete case this simple rule no longer holds, since the decision maker knows with certainty that the underlying revenue-cost-ratio dynamics will always exceed the constant optimal investment threshold at the instant the investment opportunity should be rationally exercised (this phenomenon is known as *overshooting*). Since the difference between the constant boundary of the investment region and the state of the underlying revenue-cost-ratio at the optimal investment date may be significant (depending naturally on the characteristics of the driving stochastic dynamics), the decision maker tries to prevent suboptimal waiting by lowering the required rate of return in comparison with the continuous case.

The contents of this study are as follows. In section 2 we present the considered models and state our main findings. In section 3 we illustrate our findings numerically in an explicitly parameterized setting and in a case study based on the data from the Helsinki metropolitan area. Finally, a summary and concluding comments are presented in section 4.

2 The Models

2.1 Discrete Time Model

Our main objective is to analyze the optimal timing of investment opportunities in two different settings and to analyze the difference between the resulting optimal policies in order to characterize the circumstances under which the difference between the two rules is significant. In order to accomplish this task, we first consider the discrete time model and assume that both the revenues as well as the costs associated with the particular investment opportunity evolve according to a pair of potentially correlated geometric random walks. More precisely, we assume that the revenue flow P_n is of the exponential form

$$P_n = P_0 e^{V_n},\ P_0 = p \in \mathbb{R}_+,$$

where V_n denotes the underlying driving random walk. We assume that it is characterized by the first order stochastic difference equation

$$V_{k+1} = V_k + \left(\mu_P - \frac{1}{2}\sigma_P^2\right) + \sigma_P W_{k+1}, \quad V_0 = 0,$$

where the drift coefficient $\mu_P \in \mathbb{R}$ and the volatility coefficient $\sigma_P \in \mathbb{R}_+$ are known exogenously determined parameters, and $\{W_k\}_{k\in\mathbb{N}}$ is an IID sequence of $N(0,1)$-distributed random variables. In a completely analogous way, we model the costs C_n as

$$C_n = C_0 e^{U_n},\ C_0 = c \in \mathbb{R}_+,$$

where the underlying driving random walk U_n is characterized by the first order stochastic difference equation

$$U_{k+1} = U_k + \left(\mu_C - \frac{1}{2}\sigma_C^2\right) + \sigma_C Z_{k+1}, \quad U_0 = 0. \tag{1}$$

In (1) both the drift coefficient $\mu_C \in \mathbb{R}$ as well as the volatility coefficient $\sigma_C \in \mathbb{R}_+$ are assumed to be known exogenously determined parameters and $\{Z_k\}_{k\in\mathbb{N}}$ is an IID sequence of $N(0,1)$-distributed random variables. In order to admit potential statistical dependence between revenues and costs, we assume that the normally distributed variables W_k, Z_k are correlated and assume that $\operatorname{corr}[W_k, Z_k] = \rho \in [-1, 1]$ for all k.

Having characterized the underlying stochastically fluctuating revenue and cost dynamics, we now assume that exercising the investment opportunity results into a flow of revenues which lasts for T periods from exercise after which no subsequent revenues are realized from the investment. Consequently, the expected cumulative present value of the cash flow generated by the flow of revenues reads as

$$PV_0 = \mathbb{E}_p^{\mathbb{P}} \sum_{k=1}^{T} \frac{P_k}{R_f^k} = \frac{e^{\mu_P} p}{R_f - e^{\mu_P}} \left(1 - \left(\frac{e^{\mu_P}}{R_f}\right)^T\right),$$

where R_f denotes the discount factor. Given the nature of the considered investment opportunities, it is sensible to assume that there is a lag $\delta \geq 0$ (which may be significant) between the date at which the investment opportunity is exercised and the date at which the accrual of revenues is initiated (i.e. it takes *time to build* after the particular investment decision has been exercised). Given this assumption, we find that the expected present value of future revenues at a given arbitrary exercise date k reads as

$$\mathbb{E}_{(k,P_k)}^{\mathbb{P}} \left[R_f^{-\delta} PV_{k+\delta}\right] = \mathcal{M} P_k,$$

where

$$\mathcal{M} = \frac{e^{\mu_P}}{R_f - e^{\mu_P}} \left(1 - \left(\frac{e^{\mu_P}}{R_f}\right)^T\right) \left(\frac{e^{\mu_P}}{R_f}\right)^\delta.$$

It is clear that if the discount factor R_f dominates the percentage growth rate of the flow of revenues, then the longer the delay is, the smaller is the multiplier $\mathcal{K}$ and the lower is the expected present value of the future cash flow.

The objective of a rational risk neutral value maximizing investor is to determine the date at which the maximum expected net present value of the investment opportunity is attained. More precisely, a rational investor solves the optimal timing problem

$$\tilde{V}(p,c) = \sup_N \mathbb{E}^{\mathbb{P}}_{(p,c)} \left[R_f^{-N} \left(\mathcal{M}P_N - C_N\right)^+ \right] \tag{2}$$

where N is an arbitrary stopping time. The two-dimensional optimal timing problem (2) is typically extremely difficult to be solved explicitly, if possible at all. Fortunately, in our setting the problem can be re-expressed in a simpler form which admits an explicit solution. To this end, we first present the following auxiliary Lemma.

Lemma 2.1. *The optimal timing problem* (2) *characterizing the value of the investment opportunity can be re-expressed as*

$$\tilde{V}(p,c) = c \sup_N \mathbb{E}^{\mathbb{P}}_{p/c} \left[\tilde{R}_f^{-N} \left(\mathcal{M}Q_N - 1\right)^+ \right],$$

where $\tilde{R}_f = R_f e^{-\mu_C}$ *denotes the net appreciation rate of costs,* $\Sigma^2 = \sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho = \mathrm{var}[\sigma_P W_1 - \sigma_C Z_1]$ *denotes the total variance of the difference of the driving random walks,*

$$Q_n = \frac{P_n}{C_n} = \frac{p}{c} \exp\left(\left(\mu_P - \mu_C - \frac{1}{2}\Sigma^2 \right) n + \Sigma \sum_{k=1}^{n} \xi_k \right)$$

and $\{\xi_k\}_{k\in\mathbb{N}}$ *is an IID-sequence of* $N(0,1)$*-distributed random variables.*

Proof. See Appendix A. □

Lemma 2.1 demonstrates how the original timing problem based on the potentially correlated revenue and cost processes can be reduced into a simpler timing problem of the revenue-cost-ratio process P_k/C_k. Consequently, the determination of the optimal investment timing policy can be determined solely on the basis of this one dimensional ratio which itself constitutes a standard geometric random walk. Our main results on this problem are now summarized in the following:

Theorem 2.2. *Assume that the logarithmic discount factor dominates the growth rate of the revenue flow, that is, that* $\mu_P < \ln R_f$. *Assume also that the condition* $\mu_P - \mu_C > \frac{1}{2}(\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho)$, *guaranteeing the almost sure finiteness of the optimal exercise date, is satisfied. Then the value of* *the investment opportunity reads as*

$$\tilde{V}(p,c) = \begin{cases} \mathcal{M}p - c, & p/c \geq S^* \\ \frac{p}{S^*}\mathbb{E}\left[e^{\zeta_G}; \zeta_G > \ln\left(\frac{cS^*}{p}\right)\right] - c, & p/c \leq S^*, \end{cases}$$

where $N_{S^*} = \inf\{n \geq 0 : Q_n \geq S^*\}$ *is the first time when the revenue-cost-ratio exceeds the unique optimal investment threshold* S^* *given by the identity*

$$\mathcal{M}S^* = \exp\left\{\sum_{k=1}^{\infty} \frac{R_f^{-k}}{k}\left\{e^{\mu_P k}\Phi\left(\frac{(\mu_P - \mu_C + \frac{1}{2}\Sigma^2)\sqrt{k}}{\Sigma}\right) - e^{\mu_C k}\Phi\left(\frac{(\mu_P - \mu_C - \frac{1}{2}\Sigma^2)\sqrt{k}}{\Sigma}\right)\right\}\right\},$$

where the random variable ζ_G *is given via the characteristic function*

$$\mathbb{E}\left[e^{i\eta\zeta_G}\right] = \exp\left\{\sum_{k=1}^{\infty} \frac{R_f^{-k}e^{\mu_C k}}{k}\left[\exp\left(i\eta kM - \frac{1}{2}\eta^2 k\Sigma^2\right)\Phi\left(\frac{kM}{\Sigma} + i\eta\sqrt{k}\Sigma\right) - \Phi\left(\frac{\sqrt{k}M}{\Sigma}\right)\right]\right\},$$

and $\Phi(x)$ *denotes the cumulative distribution function of the standard normal distribution.*

Proof. See Appendix B □

Theorem 2.2 characterizes explicitly the optimal investment threshold as well as the value of the optimal investment timing policy. In contrast with the standard models based on continuous dynamics, we now see that the optimal boundary has to be expressed as an infinite series. Even though its value cannot be derived explicitly, its terms converge very rapidly due to discounting and, therefore, it can be used very efficiently in numerical illustrations. It is worth noticing that the value of the optimal investment timing policy satisfies the value matching condition requiring continuity across the optimal boundary. Thus, the standard balance equation holds and the value of the investment opportunity coincides at the optimal boundary with its full costs measured as the sum of the sunk investment cost and the lost option value to wait. However, in contrast to the standard real option models of irreversible investment, the smooth pasting principle does not hold in this case. The main reason for this observation is that in the discrete case the value of the discontinuous rent-cost-ratio Q_n *exceeds with certainty* the optimal boundary S^* at the date N_{S^*} where investing becomes optimal. Put somewhat differently, from the point of view of an investor, both the date N_{S^*} at which the investment opportunity is exercised as well as the state at which exercise eventually occurs $Q_{N_{S^*}}$ are random prior exercise within a discrete-time setting. A rational investor takes this *overshooting risk* into account beforehand and lowers the required rate of return accordingly (in comparison with the standard continuous model). This adjustment then results into the non-differentiability of the value of the investment opportunity at the optimal exercise boundary.

2.2 Continuous Time Model

Having considered the discrete time model, we now present its more familiar continuous time counterpart. In this case, we assume that the revenue flow evolves according to a geometric Brownian motion process characterized by the stochastic differential equation

$$dP_t = \mu_P P_t dt + \sigma_P P_t dW_t, \quad P_0 = p,$$

where W_t is a standard Wiener process, μ_P denotes the percentage growth rate of the expected flow of revenues and $\sigma_P > 0$ denotes the volatility of the flow. Analogously, the costs associated with the investment opportunity evolve according to another geometric Brownian motion process characterized by the stochastic differential equation

$$dC_t = \mu_C C_t dt + \sigma_C C_t dZ_t, \quad C_0 = c,$$

where Z_t is a standard Wiener process, μ_C denotes the percentage growth rate of the expected investment costs and $\sigma_C > 0$ denotes the volatility of costs. In order to capture the potential statistical dependence between revenues and costs, we again assume that the driving Wiener processes are correlated with a correlation coefficient $\rho \in [-1, 1]$.

Along the lines of our previous analysis, we assume that the flow of rents generated by the exercise of the investment opportunity lasts up to an exogenously given date T from exercise. Consequently, the expected cumulative present value of the revenue flow now reads as

$$PV_0 = \mathbb{E}_p^{\mathbb{P}} \int_0^T e^{-r_f s} P_s ds = \frac{p}{r_f - \mu_P} \left(1 - e^{-(r_f - \mu_P)T}\right),$$

where r_f denotes the prevailing discount rate. In order to capture the potential delay of the initiation of the accrual of revenues after the investment opportunity has been exercised, we assume that the investor faces a lag of length δ (cf. Alvarez and Keppo (2002) and Bar-Ilan and Strange (1996)). Given this assumption, we find that the expected cumulative present value of future revenues discounted back to a given arbitrary exercise date t reads as

$$\mathbb{E}_{(t,P_t)}^{\mathbb{P}} \left[e^{-r_f \delta} PV_{t+\delta}\right] = \mathcal{K} P_t,$$

where

$$\mathcal{K} = \frac{e^{-(r_f-\mu_P)\delta}}{r_f-\mu_P}\left(1-e^{-(r_f-\mu_P)T}\right).$$

Along with our observation in the discrete time setting, we again find that if the discount rate dominates the rate at which the expected revenues grow, then the longer the delay is, the lower is the expected cumulative present value of the future revenue flow.

Having characterized the underlying return and costs dynamics, we now introduce the considered timing problem. In the present case the objective of a rational investor is to determine the date at which the maximum

$$V(p,c) = \sup_{\tau} \mathbb{E}^{\mathbb{P}}_{(p,c)}\left[e^{-r_f\tau}(\mathcal{K}P_\tau - C_\tau)^+\right] \tag{3}$$

is attained. That is, the objective of the investor is to determine a timing policy maximizing the expected net present value of the investment opportunity. It is worth noticing that this problem can be interpreted as the determination of the value and optimal exercise policy of a perpetual American exchange option written on dividend paying stock (cf. McDonald and Siegel (1986)). Our main result on this optimal timing problem is summarized in the following:

Theorem 2.3. *Assume that the discount rate dominates the growth rate of revenues, that is, that $r_f > \mu_P$. Assume also that the condition $\mu_P - \mu_C > \frac{1}{2}(\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho)$, guaranteeing the almost sure finiteness of the optimal exercise date, is satisfied. Then, the value of the optimal investment strategy reads as*

$$V(p,c) = \begin{cases} \mathcal{K}p - c & p/c \geq R^* \\ (\mathcal{K}R^* - 1)p^{\psi}c^{1-\psi}R^{*-\psi} & p/c < R^*, \end{cases} \tag{4}$$

where

$$R^* = \frac{\psi}{(\psi-1)\mathcal{K}}$$

denotes the critical exercise threshold for the revenue-cost ratio process P_t/C_t at which the investment opportunity should be optimally exercised,

$$\psi = \frac{1}{2} - \frac{\mu_P-\mu_C}{\Sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu_P-\mu_C}{\Sigma^2}\right)^2 + \frac{2(r_f-\mu_C)}{\Sigma^2}} > 1,$$

and $\Sigma^2 = \sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho$.

Proof. See Appendix C. □

Theorem 2.3 characterizes the optimal investment policy in the case where the underlying revenue and cost dynamics are modeled as continuous processes. Along the lines of our findings in the discrete case, we again find that the optimal policy satisfies the standard balance equation. However, in contrast to the discrete case, the value of the optimal policy is in this case differentiable across the exercise boundary (i.e. it satisfies the smooth fit principle). The reason for this observation is the continuity of the underlying revenue and cost dynamics. Even though the optimal exercise date is random in this case as well, the state at which the revenue-cost ratio will be at the exercise date is not.

It is at this point worth pointing out that even though both the length T of the revenue flow as well as the delay δ, measuring the time to build, affect both optimal investment thresholds S^* and R^*, it does not typically affect their ratio S^*/R^*. More precisely, if $r_f = \ln R_f$, then

$$\frac{\mathcal{M}}{\mathcal{K}} = \frac{(r_f - \mu_P)e^{\mu_P}}{e^{r_f} - e^{\mu_P}}.$$

Since this factor determines the sensitivity of the ratio between the optimal investment thresholds with respect to changes in T and δ, we observe that S^*/R^* is independent of T and δ whenever $r_f = \ln R_f$ holds.

3 Numerical Illustration of Results

3.1 Comparison

Our objective is now to numerically investigate the difference between the proposed optimal investment timing policies in order to characterize the potentially significant quantitative difference between the two different cases. In accordance with the notation in our previous section, we denote as S^* the optimal investment threshold in the discrete case and as R^* the optimal investment threshold in the continuous case. In order to characterize the relative difference between the two optimal investment thresholds, we first illustrate graphically the ratio S^*/R^* of the optimal thresholds as a function of the volatility σ_P of the revenue flow for three values of the correlation coefficient $\rho = 25\%, 0, -25\%$ under the parameter values $\mu_C = 0.01, \mu_P = 0.03, \sigma_C = 0.1, R_f = 1.04,$

$r_f = \ln R_f \approx 3.92\%, \delta = 1$, and $T = 50$. As Figure 1 indicates, the difference between the two

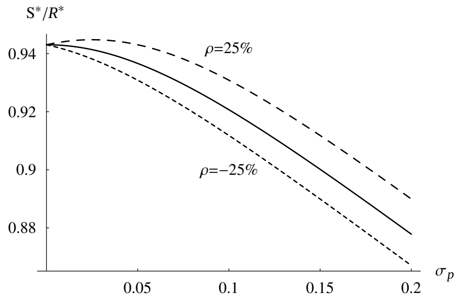

Figure 1: The ratio S^*/R^* as function of σ_P

optimal thresholds increases as volatility increases. Consequently, the larger the volatility of the cash flow is, the greater is the difference between the two optimal timing rules.

The numerical difference between the optimal thresholds as functions of the cost volatility σ_C is illustrated in the following table in the case where $\mu_C = 0.01, \mu_P = 0.03, \sigma_P = 0.05, \rho = 0, R_f = 1.04$, $r_f = \ln R_f \approx 3.92\%, \delta = 1$, and $T = 50$. Table 1 shows that both thresholds are increasing as

σ_C	0.05	0.1	0.15	0.2	0.25	0.3
S^*	0.0844	0.0945	0.1103	0.1307	0.1552	0.1834
R^*	0.088	0.1009	0.1209	0.1474	0.18	0.2189
$\frac{R^*}{S^*}$	1.0426	1.0677	1.0967	1.1277	1.1601	1.1937

Table 1: The optimal thresholds as functions of cost volatility σ_C

functions of the cost volatility parameter σ_C. However, as it is also clear from our numerical results, the difference between the two thresholds is increasing as function of σ_C as well. This once again emphasizes the potentially significant difference between the two optimal timing rules.

3.2 A case study using data from Helsinki

Data from Helsinki metropolitan area (HMA) in Finland is employed in an empirical case study to examine the difference between the required rates of returns implied by the continuous and discrete time models[2]. With a large number of vacant lots zoned for housing and relatively reliable data available, HMA is an attractive area for the case study. All the indices used are on a quarterly basis and the full sample period is 1988Q1-2008Q4.

A net rental price index is constructed to estimate the development of cash flows generated by rental housing. The (gross) rental price series represent the average rent per square meter in privately financed rental dwellings in HMA. The rental price data are on an annual basis until 2002. Thus, while the annual growth rate is based on the actual rental price series, the quarterly changes are approximated according to the "living, heating and light" part (1989-1999) and to the "rental cost" part (2000-2002) of the nationwide cost of living index. From 2003 onwards a quarterly rental price index is utilized.

To get the net rental cash flow, an approximation of the maintenance costs of housing is subtracted from the rental price level. The base value for the proxy for the maintenance costs is the average per square meter maintenance cost of privately financed flats in the HMA in 2007. The variation in maintenance costs over time is calculated based on the multi-storey housing (flats) section of the property maintenance cost indices[3]. The index represents national level figures. Nevertheless, in a relatively small and coherent country such as Finland the national index represents well the development in HMA.

The evolution of construction costs faced by the developer is approximated by the tender price index for new housing construction in HMA. The index includes changes in productivity, in the price of inputs as well as in the profit margins of the construction companies. The tender price index is available only since 1988Q1, which limits the length of the sample period. All the data utilized in the case study are provided by Statistics Finland, except for the tender price index which is produced by Rapal ltd. The net rental cash flow and construction cost indices are deflated by the consumer price index and natural logs are taken from all the deflated indices. The indices are shown in Figure 1 and descriptive statistics of the differenced series are presented in Table 1. Since a rental market deregulation took place in Finland in several stages during 1992-1995, descriptive statistics

[2]HMA, as defined here, consists of Helsinki and the three nearest surrounding municipalities Espoo, Kauniainen and Vantaa.

[3]Since most of the rental dwellings in HMA are flats in multi-storey buildings, this index should approximate the evolution of the rental housing maintenance costs well.

based on two different sample periods are reported: the full sample period over 1988Q1-2008Q4 and a shorter sample period from 1996Q1 to 2008Q4.

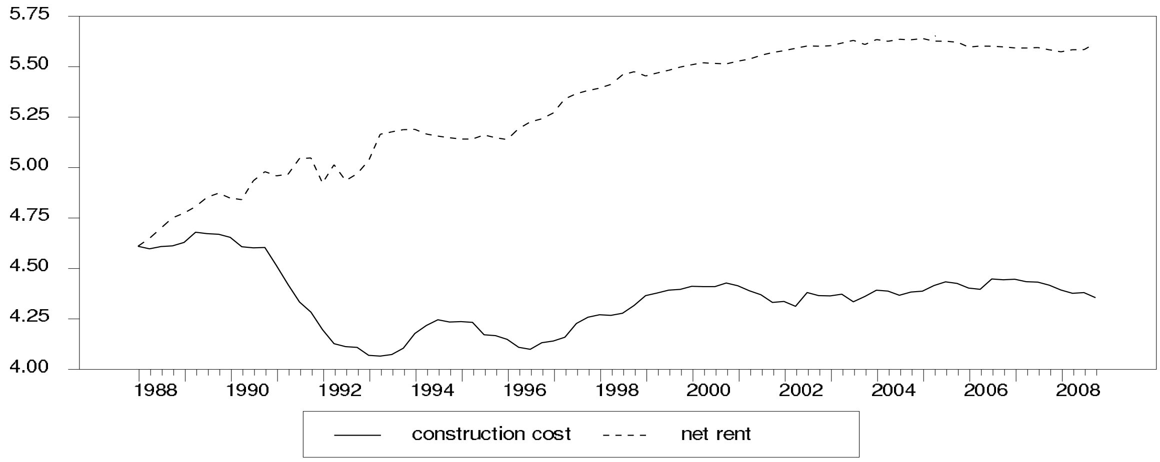

Figure 2: Real net rental price and construction cost indices

The real net rental cash flows have substantially increased during the last two decades, whereas the construction costs faced by a developer have declined. During the last few years, however, the rental index has slightly declined. The recent development can be attributed to both lower rental price growth and to faster growth of the maintenance costs.

In 1996 the tender price index was at a very low level due to the severe recession in the Finnish economy in the early and mid 1990s. On the other hand, in 1988 the tender price index was at its top in real terms. Over the whole sample period, there does not seem to be a trend in the construction cost index[4]. Hence, it is reasonable to assume that in the long term the construction cost growth rate equals the overall rate of inflation, i.e. that the average real construction cost growth is zero. The growing trend in the real net rental cash flows, in turn, is in accordance with the implications of the basic urban economic theory regarding a rapidly growing metropolitan area such as HMA.

At the quarterly level, there does not appear to be any notable correlation between the growth rates. The correlation has not notably altered after the rental market deregulation. There are some differences between the periods, however. Both growth rates have been less volatile after 1996.

[4]When the construction cost growth is regressed on a constant and a trend, the coefficient for the trend is -.0003 with a standard error of .0007.

Variable	Geometric Mean (Annualized)	Standard Deviation (Annualized)	Jarque-Bera (p-value)	1^{st} order autocorrelation
Net rental cash flows 1988Q1-2008Q4	.048	.067	.000	.004
Construction costs 1988Q1-2008Q4	-.012	.067	.030	$.515^{**}$
Net rental cash flows 1996Q1-2008Q4	.036	.035	.000	$.305^{*}$
Construction costs 1996Q1-2008Q4	.014	.050	.151	.190

Correlation coefficients between the growth rates:

1988Q1-2008Q4: .014

1996Q1-2008Q4: -.020

*denotes statistical significance at the 1% level. The autocorrelation figures are based on an AR(1) model with a deterministic constant.

Table 2: Descriptive statistics of the growth rates

Regarding construction costs, this is not likely to be due to the rental market liberalization, but rather because of the housing market boom-bust period during the late 1980s and early 1990s that substantially affected the profit margins of the construction sector. Furthermore, rental cash flows appear to have been highly autocorrelated after the rental market deregulation. The absence of autocorrelation prior to the deregulation is unsurprising, since the real rental price growth was to a large extent determined by the inflation rate and changes in the rental ceilings.

Because of the complications with the pre 1996 period, the case study concentrates on the 1996Q1-2008Q4 data. Note also that the prices of new rental contracts are typically more volatile than the prices of the whole rental housing stock. Therefore, the $S^*/R^* = 0.9654$ ratio estimated in this case study is likely to be greater than the actual one, since the general rental price level is used in the analysis[5]. Nevertheless, even the figure 0.9654 is economically significantly different from one. If the future volatility of real rental cash flows was similar to the whole 1988Q1-2008Q4 period, the S^*/R^*-ratio would be as low as 0.8526.

[5]Unfortunately, sufficiently long data on new rental contracts is not available.

4 Conclusions

We investigated how the discreteness of the underlying revenue and cost dynamics affect the optimal timing of real estate investment opportunities in comparison with continuous time models. We showed that the standard continuous time real option model of irreversible investment leads to slower investment timing than the discrete time model. According to our findings this result is based on the fact that, in comparison with the discrete time case, continuous time models overestimate the required rate of return of an investment opportunity. Since this difference may be significant and increases as volatility becomes higher, decisions based on continuous time models may result in significantly slower exercise of investment opportunities. Thus, one interesting real estate investment interpretation of our results is that the use of continuous time models leads to tinyer and suboptimal amount of housing supply in comparison with the discrete time model.

Our approach is essentially based on the idea that in an illiquid market irreversible investment decisions should be based on a discrete time model of the underlying cost and revenue processes due to the special characteristics of such market. Naturally, this is only one possible approach for analyzing the optimal timing of investment decisions. An alternative approach would be to model the underlying costs and revenues as standard continuous-time processes and impose constraints on how these variables are observed in time (i.e. how information on the underlying variables is updated). This is an interesting and challenging question which is at present out of the scope of our study and, thus left for future research.

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A Proof of Lemma 2.1

Proof. Let (W, Z) be a Gaussian random vector with standard normal marginal distributions W and Z for which $\mathrm{corr}[W, Z] = \rho$. Using the marginals W and Z, define the IID sequences $W, W_1, W_2, \dots$ and $Z, Z_1, Z_2, \dots$ and, consequently, the general random walks V and U via the recursions $V_{k+1} = V_k + \left(\mu_P - \frac{1}{2}\sigma_P^2\right) + \sigma_P W_k$ and $U_{k+1} = U_k + \left(\mu_C - \frac{1}{2}\sigma_C^2\right) + \sigma_C Z_k$. Finally, denote as P and C the geometric random walks defined as

$$P_n = P_0 e^{V_n}, \ P_0 = p \in \mathbb{R}_+, \text{ and } C_n = C_0 e^{U_n}, \ C_0 = c \in \mathbb{R}_+.$$

Define the equivalent measure $\tilde{\mathbb{P}}$ by the likelihood ratio

$$\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}} = \exp\left(\sigma_C U_n - \frac{1}{2}\sigma_C^2 n\right).$$

It is well known that the random variables $\tilde{Z}_i := Z_i - \sigma_C$ are independent and standard normal under the measure $\tilde{\mathbb{P}}$. Therefore we find that

$$C_n = c \exp\left(\left(\mu_C + \frac{1}{2}\sigma_C^2\right) n + \sigma_C \sum_{k=1}^n \tilde{Z}_k\right).$$

Analogously,

$$P_n = p \exp\left(\left(\mu_P - \frac{1}{2}\sigma_P^2 + \sigma_P\sigma_C\rho\right) n + \sigma_P \sum_{k=1}^n \tilde{W}_k\right),$$

where $\rho = \mathrm{corr}[W, Z]$ and the random variables $\tilde{W}_i := W_i - \sigma_C\rho$ are independent and standard normal under the measure $\tilde{\mathbb{P}}$. Define the quotient process Q as $Q_n = \frac{P_n}{C_n}$ and let $\Sigma^2 = \sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho = \mathrm{var}[\sigma_P W_1 - \sigma_C Z_1]$ denote the variance of the difference $\sigma_P W_1 - \sigma_C Z_1$. Then for any $n \geq 1$

$$Q_n = \frac{p}{c} \exp\left(\left(\mu_P - \mu_C - \frac{1}{2}\Sigma^2\right) n + \sigma_P \sum_{k=1}^n \tilde{Z}_k - \sigma_C \sum_{k=1}^n \tilde{W}_k\right).$$

Since the random vector $(\tilde{W}, \tilde{Z})$ is Gaussian, the process Q can be written as

$$Q_n = \frac{p}{c} \exp\left(\left(\mu_P - \mu_C - \frac{1}{2}\Sigma^2\right) n + \Sigma \sum_{k=1}^n \xi_k\right),$$

where the sequence ξ_i is independent and standard normal under the measure $\tilde{\mathbb{P}}$.

Consider the optimal stopping problem (2). Using the definition of the process Q and the measure $\tilde{\mathbb{P}}$, the value $\tilde{V}$ can be rewritten as

$$\tilde{V}(p, c) = \sup_N \mathbb{E}^{\mathbb{P}}_{(p,c)}\left[R_f^{-N}\left(\mathcal{M}Q_N - 1\right)^+ C_N\right] = c \sup_N \mathbb{E}^{\tilde{\mathbb{P}}}_{p/c}\left[R_f^{-N} e^{\mu_C N}\left(\mathcal{M}Q_N - 1\right)^+\right].$$

Define now the adjusted discount factor $\tilde{R}_f = R_f e^{-\mu_C}$. Now, the optimal stopping problem can be rewritten in the form

$$V(p,c) = c \sup_N \mathbb{E}^{\mathbb{P}}_{p/c}\left[\tilde{R}_f^{-N}\left(\mathcal{M}Q_N - 1\right)^+\right].$$

$\square$

B Proof of Theorem 2.2

Proof. Let Γ be a geometrically distributed random variable with $\mathbf{P}(\Gamma > k) = \tilde{R}_f^{-k}$ and independent of the random variables W and Z. Define the random variable G as

$$G := \inf\left\{k = 0, 1, \ldots, \Gamma : \zeta_k = \max_{j=0,1,\ldots,\Gamma} \zeta_j\right\},$$

where the process ζ is defined as $\zeta_n = Mn + \Sigma\sum_{k=1}^n \xi_k$ with $M = \mu_P - \mu_C - \frac{1}{2}\Sigma^2$ and $\zeta_0 = 0$. In other words, random variable G is defined as the historical maximum of the random walk ζ up to the independent, geometrically distributed random time Γ. It is now easy to check that the assumption $\mu_P < \ln R_f$ implies that $\phi(1) < \tilde{R}_f$, where ϕ is the moment generating function of a $N(M, \Sigma^2)$-random variable. Under this condition, the results of Darling et al (1972), pages 1367 – 1368, imply that the optimal investment policy is a threshold policy with unique finite threshold

$$S^* = (1/\mathcal{M})\mathbb{E}\left[e^{\zeta_G}\right] = (1/\mathcal{M})\exp\left\{\sum_{k=1}^{\infty}\frac{\tilde{R}_f^{-k}}{\sqrt{2\pi\Sigma^2 k^3}}\int_0^{\infty}(e^z - 1)\exp\left(-\frac{(z - Mk)^2}{2k\Sigma^2}\right)dz\right\}.$$

To establish the claimed expression on S^*, it is a matter of straightforward calculus to establish that for all $k \geq 1$,

$$\frac{1}{\sqrt{2\pi\Sigma^2 k}}\int_0^{\infty}(e^z - 1)\exp\left(-\frac{(z - Mk)^2}{2k\Sigma^2}\right)dz = e^{(\mu_P - \mu_C)k}\cdot\Phi\left(\frac{(M + \Sigma^2)\sqrt{k}}{\Sigma}\right) - \Phi\left(\frac{M\sqrt{k}}{\Sigma}\right),$$

where $\Phi(x)$ denotes the cumulative distribution function of the standard normal distribution. On the other hand, the condition $M = \mu_P - \mu_C - \frac{1}{2}\Sigma^2 > 0$ is a sufficient condition for the almost sure finiteness of any first exit time of the type $N_y = \inf\{n \geq 0 : \zeta_n \geq y\}$ (cf. Feller (1971), pp. 396–397). Since the proposed optimal policy belongs to this class, we find that condition $M > 0$ guarantees the almost sure finiteness of investment timing as well.

In order to prove the claim on the optimal value function $\tilde{V}$, we find that the results of Darling et al (1972), pages 1367 – 1368, imply that that $\tilde{V}$ can be re-expressed, in terms of the random variable ζ_G as

$$\tilde{V}(p,c) = c\mathcal{M}\frac{\mathbb{E}\left[(\frac{p}{c}e^{\zeta_G} - (1/\mathcal{M})\mathbb{E}\left[e^{\zeta_G}\right])^+\right]}{\mathbb{E}\left[e^{\zeta_G}\right]}.$$

After some basic manipulations of mathematical expectation we find that $\tilde{V}$ can be re-written as

$$\begin{aligned}\tilde{V}(p,c) &= c\mathbb{E}\left[\frac{p}{cS^*}e^{\zeta_G} - 1; \zeta_G > \ln\left(\frac{cS^*}{p}\right)\right] \\ &= \begin{cases}\mathcal{M}p - c, & p/c \geq S^* \\ \frac{p}{S^*}\mathbb{E}\left[e^{\zeta_G}; \zeta_G > \ln\left(\frac{cS^*}{p}\right)\right] - c, & p/c \leq S^*.\end{cases}\end{aligned}$$

Finally, the claimed characterization of the random variable ζ_G follows from Kyprianou (2007), Theorem 1, after a straightforward calculation, i.e,

$$\begin{aligned}\mathbb{E}\left[e^{i\eta\zeta_G}\right] &= \exp\left\{\sum_{k=1}^{\infty}\frac{\tilde{R}_f^{-k}}{\sqrt{2\pi\Sigma^2k^3}}\int_0^{\infty}(e^{i\eta z} - 1)\exp\left(-\frac{(z-Mk)^2}{2k\Sigma^2}\right)dz\right\} \\ &= \exp\left\{\sum_{k=1}^{\infty}\frac{\tilde{R}_f^{-k}}{k}\left[\exp\left(i\eta kM - \frac{1}{2}\eta^2k\Sigma^2\right)\Phi\left(\frac{kM}{\Sigma} + i\eta\sqrt{k}\Sigma\right) - \Phi\left(\frac{\sqrt{k}M}{\Sigma}\right)\right]\right\}.\end{aligned}$$

□

C Proof of Theorem 2.3

Proof. In order to simplify the real option valuation problem (3), we first notice that the cost process can be expressed in the form $C_t = ce^{\mu_C t}\mathcal{M}_t$, where

$$\mathcal{M}_t = e^{\sigma_C Z_t - \frac{1}{2}\sigma_C^2 t}$$

is a positive exponential martingale. Consider now the expected net present value of the investment opportunity at any future date T, that is, consider the value of the European exchange option

$$F^T(p,c) = \mathbb{E}^{\mathbb{P}}_{(p,c)}\left[e^{-r_f T}(\mathcal{K}P_T - C_T)^+\right]$$

written on dividend paying stock (cf. Fischer (1978) and Margrabe (1978)). It is now clear that

$$F^T(p,c) = c\mathcal{K}\mathbb{E}^{\mathbb{P}}_{(p,c)}\left[e^{-(r_f-\mu_C)T}\mathcal{M}_T\left(\frac{P_T}{C_T} - \frac{1}{\mathcal{K}}\right)^+\right].$$

Defining the equivalent measure $\mathbb{Q}$ by the likelihood ratio $d\mathbb{Q}/d\mathbb{P} = \mathcal{M}_T$ then implies that the expected net present value of the investment opportunity at any future date T can be re-expressed as

$$F^T(p,c) = c\mathcal{K}\mathbb{E}^{\mathbb{Q}}_{(p,c)}\left[e^{-(r_f-\mu_C)T}\left(\frac{P_T}{C_T} - \frac{1}{\mathcal{K}}\right)^+\right],$$

where under the measure $\mathbb{Q}$

$$dP_t = (\mu_P + \sigma_P\sigma_C\rho)P_t dt + \sigma_P P_t d\hat{W}_t, \quad P_0 = p$$

and

$$dC_t = (\mu_C + \sigma_C^2)C_t dt + \sigma_C C_t d\hat{Z}_t, \quad C_0 = c.$$

Defining now the rent-cost ratio process as $R_t = P_t/C_t$ shows that

$$dR_t = (\mu_P - \mu_C)R_t dt + \sigma_P R_t d\hat{W}_t - \sigma_C R_t d\hat{Z}_t, \quad R_0 = p/c.$$

Consequently, the expected net present value of the investment opportunity at any future date T can be expressed as

$$F^T(p,c) = c\mathcal{K}\mathbb{E}^{\mathbb{Q}}_{p/c}\left[e^{-(r_f-\mu_C)T}\left(R_T - \frac{1}{\mathcal{K}}\right)^+\right].$$

Applying this observation to our original timing problem shows that

$$V(p,c) = c\mathcal{K}\sup_{\tau}\mathbb{E}^{\mathbb{Q}}_{p/c}\left[e^{-(r_f-\mu_C)\tau}\left(R_\tau - \frac{1}{\mathcal{K}}\right)^+\right].$$

It is now a standard exercise in one-dimensional optimal stopping theory to demonstrate that if the condition $r_f > \mu_P$, guaranteeing the convergence of the present value, and the condition $\mu_P - \mu_C > \frac{1}{2}(\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho)$, guaranteeing the almost sure finiteness of the optimal exercise date, are satisfied then the value of the optimal policy reads as (4) and R^* constitutes the threshold at which the opportunity should optimally be exercised. $\square$

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ISSN 1796-3133