Marja-Liisa Halko
and Juha Virrankoski
Coordination in the Labor Market

Aboa Centre for Economics

Discussion Paper No. 64
Turku 2011

Copyright © Author(s)

ISSN 1796-3133

Printed in Uniprint
Turku 2011

Marja-Liisa Halko and Juha Virrankoski
Coordination in the Labor Market

Aboa Centre for Economics
Discussion Paper No. 64
February 2011

ABSTRACT

We solve the equilibrium market structure in a labor market where vacancies and unemployed workers can meet either in an intermediated market where wages are determined by take-it-or-leave-it offers, or in a directed search market where firms post wages. By using an intermediary agents avoid the coordination problem which prevails in the search market. We study a monopolistic intermediary and perfect competition between intermediaries, and we consider the welfare properties of an intermediary institution, compared to an economy with an uncoordinated search process only.

JEL Classification: D40, J41, J64

Keywords: intermediary, matching, labor market

Contact information

Marja-Liisa Halko

Department of Accounting and Finance, Aalto University School of Economics. E-mail: marja-liisa.halko@hse.fi

Juha Virrankoski

Corresponding author. Department of Economics, University of Turku. E-mail: juha.virrankoski@tse.fi

Acknowledgements

We thank participants in EEA 2009 and EALE 2009 conferences, and participants in PCRC seminar in Turku, especially Hannu Salonen and Heikki Pursiainen, for constructive comments. We are also grateful to Yrjö Jahnsson Foundation for financial support.

1 Introduction

Unemployment persists in equilibrium partly because there is friction in the meetings between vacancies and unemployed job seekers. One of the sources of friction is lack of coordination in search activity between job seekers, leading to random distribution of job applications. Not all the job-seeking activity takes place in a decentralized search market: some of the employment relationships are formed by using intermediaries (private or government agencies). The services of public employment agencies are mainly free for job seekers as well as for employers. They offer also special services to firms for a commission fee. Private employment agencies typically charge firms a commission fee if they can fill the vacancy but offer their services to workers without charge. The most important roles of intermediaries according to Spulber (1996a) are "setting prices and clearing markets; providing liquidity and immediacy; coordinating buyers and sellers; and guaranteeing quality and monitoring performance".

We study a labor market where an intermediary (either a monopolistic or competitive) coordinates the matching of vacancies and unemployed workers. In equilibrium it provides immediacy to vacancies. Firms and workers participate in a coordinated market (use the services of the intermediary) or in a search market. In the coordinated market an intermediary pairs vacancies and workers and charges a commission fee from vacancies if it can allocate them a worker. Workers can use intermediary's services for free. Wages are determined by take-it-or-leave-it offers made by either firms or workers. This is a natural choice in bilateral meetings when both parties have an option to go to the search market. The rest of the matches form in a search market where vacancies post wages publicly, and job seekers choose which vacancy to contact, without coordinating their choices.

The main contribution of this article is that we solve the equilibrium market structure as a function of unemployment-vacancy ratio. In equilibrium not all the vacancies choose the coordinated market and consequently, some of the meetings take place in a search market. Therefore the intermediary cannot extract all the available rents (see also Gehrig, 1993, p. 107). For unemployment-vacancy ratios smaller (larger) than one, the larger (smaller) the ratio the larger the relative size of the coordinated market. We show that introducing an intermediary into a search market increases the number of matches, and it benefits the short side of the market and hurts the long side. The reason for the latter is that the unemployment-vacancy ratio in the search market (which acts as an outside option) changes in favor of the short side, and the improvement of the outside option improves the position of the short side in the coordinated market. If the number of firms is determined by zero-profit condition, the equilibrium market structure is non-monotone in the firms' entry cost.

Intermediaries have been studied in two different roles: as market makers and match-makers. Market makers buy commodities from sellers and resell them to buyers (e.g. retailers, used car dealers). In Rubinstein and Wolinsky (1987) there is an endogenous number of market makers, and buyers and sellers are homogeneous. An intermediated market and a search market coexist because matching probability in the former is assumed to be higher than in the latter. Gehrig (1993) shows that when a monopolistic market maker sets bid and ask prices, agents with large gains from trade deal with the intermediary, and agents with low gains from trade go to search market. Yavas (1996) generalizes Gehrig's model by endogenizing search intensities, allowing traders to go to the intermediary if they fail to trade in the search market, and assuming that the intermediary has an ability to provide immediate service. As a result, buyers and sellers with high or low gains from trade use the intermediary whereas agents with moderate gains from trade choose the search market. Spulber (1996b) studies a dynamic model with heterogeneous consumers and suppliers and price setting market makers. He compares the market equilibrium in the case of competing market makers with the equilibrium in the traditional supply and demand model. The basic reason for a positive number of active market makers in equilibrium is that they can decrease the time cost in searching. In Shevchenko (2004) there is an endogenous number of middlemen who choose the number of goods they store. Biglaiser (1993) emphasizes intermediary's incentive to invest in becoming an expert on goods he sells because he buys more goods than an individual buyer. Concerns about reputation gives an intermediary a larger incentive (compared with sellers) to give buyers correct information about the quality of goods. Miao (2006) considers a model where heterogenous buyers and homogenous sellers choose between a centralized market where market makers publicly post bid and ask prices, and a decentralized market where the terms of trade are determined by Nash bargaining. Opening a centralized market does not necessarily improve social welfare since trading in there is assumed to have a cost, and it makes the decentralized market tighter which makes buyers there worse off.

Matchmakers do not trade but they match the trading partners (e.g. employment and matrimonial agencies, real estate brokers). This is the setting in the present article. In Yavas (1994) heterogeneous buyers and sellers choose search intensities, and a monopolistic matchmaker sets a commission fee which is proportional to trading surpluses. He shows that buyers with low valuation and sellers with high valuation choose the matchmaker, and the other agents choose the search market. Bloch and Ryder (2000) study especially the effects of two different pricing schemes for the matchmaker. When the matchmaker charges a uniform fee from all agents, only the agents with high gains from trade choose the intermediated market. On the other hand, when the fee is proportional to the trading surplus, only the agents with low gains from trade choose the intermediated market.

The rest of the paper is organized as follows. In Section 2 we consider a model with a fixed number of firms and workers, and we solve the market structure - the proportion of firms that chooses the coordinated market - as a function of unemployment-vacancy ratio θ and the offer-making probability α. In Section 3 we analyze a model where the number of firms is determined by free entry. In Section 4 we consider a model where there is Bertrand competition between intermediaries. Section 5 concludes. The Appendix contains verifications of the second-order conditions for market structure.

2 Monopolistic Intermediary and Fixed Number of Firms

The economy is populated by a large number u of unemployed workers, a large number v of vacancies, and an intermediary. Let $\theta \equiv u/v$. Each matched firm-worker pair produces a unit output. We consider a static model[1] where agents make their choices in the following stages: (1) The intermediary announces fee f it charges from firms which manage to hire an applicant. (2) Fraction ρ of the firms register with the intermediary, that is, they choose the coordinated market. (3) All workers go to the intermediary who forms as many firm-worker pairs as possible. (4) The firms who did not choose the coordinated market go to search market and post a wage. (5) In the coordinated market, a firm makes a take-it-or-leave-it offer with probability $\alpha \in [0, 1]$, and the worker makes an offer with probability $1 - \alpha$. If the respondent rejects the offer, the firm and the worker go to the search market. (6) The workers who were left without a partner in the coordinated market and those who did not reach an agreement in the coordinated market search among the firms who posted wages. (7) Matches are formed in the search market, and firms pay the wage they posted.

In analyzing the search market we thus utilize the standard urn-ball model where vacancies represent the urns and unemployed represent the balls. Suppose that the vacancies are identical and the unemployed observe them all. Each of the unemployed chooses one vacancy at random. The number of unemployed that chooses a given vacancy is then binomially distributed. We assume that u and v are very large, and we approximate the binomial distribution by Poisson distribution. Denoting the unemployment-vacancy ratio in the search market by σ, the matching probability for a firm is $1 - e^{-\sigma}$, and for a worker it is $(1 - e^{-\sigma})/\sigma$. When making offers in the coordinated market the agents take into account the possibility that the respondent rejects the offer and both go to the search market. The value of this option determines the equilibrium offers.

We solve the market structure - the proportion of firms that chooses the coordinated market - as a function of unemployment-vacancy ratio θ and the offer-making probability α. We also examine the efficiency of the intermediary institution, and the utilities accruing to agents, and we compare them to an economy without an intermediary.

Consider first the outcomes in the search market. They are determined by unemployment-vacancy ratio σ in that market, which depends on the overall unemployment ratio θ and the fraction of firms which choose the coordinated market. But first we take the value of σ as given. In the search market each firm posts a wage w_s in order to attract workers, taking all other firms' wages as given. The unemployed observe all the wages and choose firms. A larger wage decreases ex post profit but it attracts more applicants. We consider

[1] One could consider a dynamic model where agents remain in the market until they find partners, and employer-employee relationships break down with some probability. This is likely to be a tedious exercise, and we doubt whether it brings any new insights compared to a static model.

a symmetric Nash equilibrium in wages. Following Kultti (1999)[2] we have

$$w_s = \frac{\sigma e^{-\sigma}}{1 - e^{-\sigma}}. \tag{1}$$

The probability for a worker to be hired in the search market is $(1 - e^{-\sigma})/\sigma$, and the expected utility U_s of a worker in the search market is $w_s(1 - e^{-\sigma})/\sigma$, that is,

$$U_s = e^{-\sigma}. \tag{2}$$

The expected utility of a firm in the search market is $V_s = (1 - e^{-\sigma})(1 - w_s)$ where the first term is the probability that a firm meets at least one worker. Using (1) we have[3]

$$V_s = 1 - e^{-\sigma} - \sigma e^{-\sigma}. \tag{3}$$

The value of σ is determined by θ and $\rho \in (0,1)$ which is the fraction of firms which choose the coordinated market. Assume first that $\theta \geq 1$. This implies that $\rho v \leq u$, and the intermediary assigns a worker to all the firms in the coordinated market. The numbers of workers and firms who go to the search market are $u - \rho v$ and $(1-\rho)v$, respectively. The ratio of workers and firms is thus equal to

$$\sigma = \frac{\theta - \rho}{1 - \rho}. \tag{4}$$

If $\theta > 1$, then $\sigma > \theta$.

If $\theta < 1$, we consider two cases. In the first case $\rho > \theta$, in other words, $\rho v > u$. All workers meet a firm in the coordinated market but some firms fail to meet a worker. Suppose a firm and a worker meet, and either of them makes a take-it-or-leave-it offer. If it is rejected, they go to the search market. As in the case where $\theta > 1$ we assume that only one agent deviates by making an offer which the respondent rejects. In the search market there is now one worker and many vacancies - those which chose the search market in the first place and those which failed to meet a worker in the coordinated market. Then the equilibrium wage the firms offer is equal to one, and therefore $V_s = 0$. Also, $V_c = -f$ because the only wage offer a worker would accept in the coordinated market is equal to one. But then no firm would choose the coordinated market. We conclude that if $\rho > \theta$, the coordinated market does not exist. For now on, we assume that $\rho < \theta$. This implies that by going to coordinated market a vacancy guarantees itself an immediate match with a worker, as in the case where $\theta > 1$. The job seeker - vacancy ratio in the search market is given by (4), but now $\sigma < \theta$.

In the coordinated market either a firm or a worker makes a take-it-or-leave-it offer. We can think that in every meeting the proposer is drawn at random, or that proportion α of firms always propose and proportion $1-\alpha$ of firms let the worker propose. If a proposal

[2] The idea of the proof is to consider a subset of firms that deviates by posting w_s' instead of w_s posted by all other firms, and a subset of workers who choose the deviating firms. Then we let the sizes of the subsets to approach zero.

[3] The same utilities result if workers send applications and make wage demands with a mixed strategy (Halko, Kultti and Virrankoski, 2008).

is rejected, we assume that both of the agents go to the search market. Suppose that a single firm deviates and makes an offer which the worker rejects. Because the number of both agents in the search market is large and this pool is augmented by only one worker and one firm, the Poisson parameter σ in the search market remains unchanged. The expected value of a firm is then $\hat{V}_c = -f + 1 - e^{-\sigma} - \sigma e^{-\sigma} < V_s$. The worker does not accept an offer lower than his outside option, $e^{-\sigma}$, and the firm does not accept an offer lower than its outside option, $1 - e^{-\sigma} - \sigma e^{-\sigma}$. The firm's expected value is thus

$$\begin{aligned} V_c &= -f + \alpha\left(1 - e^{-\sigma}\right) + (1-\alpha)\left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right) \\ &= -f + 1 - (1 + (1-\alpha)\sigma)\, e^{-\sigma}. \end{aligned} \tag{5}$$

If the firm proposes, it offers the worker $e^{-\sigma}$ which is acceped, and if the worker proposes, he offers the firm $1 - e^{-\sigma} - \sigma e^{-\sigma}$ which it accepts. If both markets exist in equilibrium, then $V_c = V_s \Rightarrow f = \alpha\sigma e^{-\sigma}$.

The expected utility of a worker is

$$\begin{aligned} U &= \frac{\rho}{\theta}\left(\alpha e^{-\sigma} + (1-\alpha)\left(1 - \left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right)\right)\right) + \left(1 - \frac{\rho}{\theta}\right) U_s \\ &= \left(\frac{\rho\sigma(1-\alpha)}{\theta} + 1\right) e^{-\sigma}, \end{aligned} \tag{6}$$

where ρ/θ is the probability that the worker is hired in the coordinated market. If the worker fails to be hired in there, he goes to the search market and receives utility U_s.

The intermediary maximizes profit $\pi = \rho v f$ by choosing f conditional on $V_c = V_s$. By (4) and $f = \alpha\sigma e^{-\sigma}$, choosing f is equivalent to choosing ρ. Further, maximizing $\rho v f$ is equivalent to maximizing ρf because v is given. The profit maximization problem is

$$\underset{\rho}{Max}\left(\frac{\rho\alpha(\theta-\rho)}{1-\rho} e^{\dfrac{\rho-\theta}{1-\rho}}\right). \tag{7}$$

In Figure 1, curve $\pi = \rho v f$ is the intermediary's isoprofit curve. The further right it lies the larger the profit. The other curve is the vacancies' indifference curve given by $V_c = V_s$. If the intermediary chooses from the left of the indifference curve, vacancies are strictly better off by choosing the coordinated market, and the intermediary can increase f. If the intermediary chooses from the right of the indifference curve, vacancies are strictly better off in the search market.

Proposition 1 *(i) If $\theta < 1$, the equilibrium market structure satisfies* $\theta = \frac{1}{2\rho}\left(1 + \rho^2 - (1-\rho)\sqrt{1 + 2\rho - 3\rho^2}\right)$ *,(ii) If $\theta > 1$, the equilibrium market structure satisfies* $\theta = \frac{1}{2\rho}\left(1 + \rho^2 + (1-\rho)\sqrt{1 + 2\rho - 3\rho^2}\right)$*,*

Proof. The first-order condition of (7) is $\theta - \rho\theta^2 - 2\rho + 2\rho^2 + \theta\rho^2 - \rho^3 = 0$, which has two solutions. Appendices A1 and A2 verify that the second-order condition is satisfied. ■

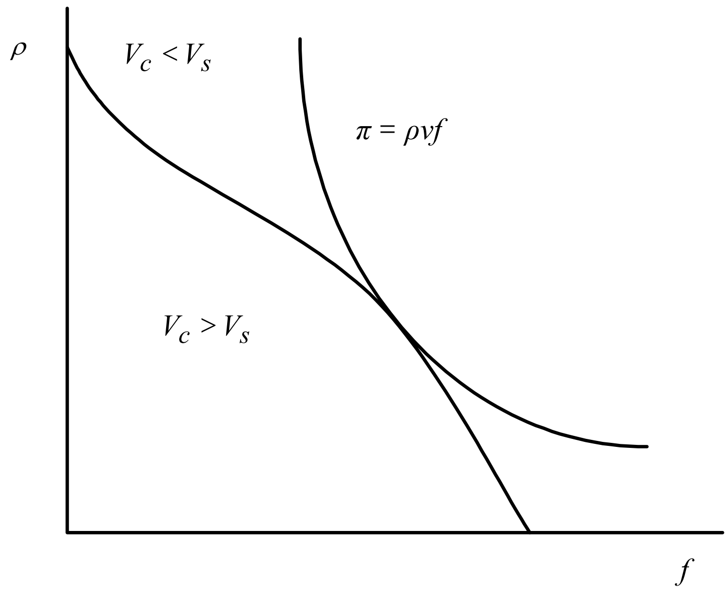

Figure 1: The intermediary's profit maximization subject to vacancies' indifference between coordinated market and search market

The fraction of firms that participates in the coordinated submarket is increasing in θ if $\theta < 1$, and it is decreasing in θ if $\theta > 1$. Differentiating V_c and V_s with respect to θ, keeping ρ and c constant, results in $\partial V_c/\partial\theta = e^{-\sigma}/\left(1-\rho\right)$ and $\partial V_s/\partial\theta = \sigma e^{-\sigma}/\left(1-\rho\right)$. If $\sigma < 1$, which holds if $\theta < 1$, then $\partial V_s/\partial\theta < \partial V_c/\partial\theta$. The equality of the utilities is restored only if there are less firms in the search market and more in the coordinated market, that is, if ρ increases. If $\sigma > 1$, which holds if $\theta > 1$, then $\partial V_s/\partial\theta > \partial V_c/\partial\theta$. The equality of the utilities is restored only if ρ decreases.

If $\theta = 1$, Proposition 1 implies that $\rho = 1$. If an offer is rejected, both go to the search market where they would be the only agents. The worker will accept any nonnegative wage the firm posts. Therefore $V_s = 1$. Also, in the coordinated market the worker will accept any non-negative wage. Then $V_c = -f + 1$. If the intermediary charges $f > 0$, the firm goes to the search market right away and gets $V_s = 1$ because there would be one worker, the one who did not match in the coordinated market because it lacks this particular firm. Therefore $f = 0$ if $\rho = 1$. If $\theta = 1$, and $\rho \to 1$, then both markets exist and the intermediary makes a positive profit.

Figure 2 summarizes how the equilibrium market structure depends on the unemployment-vacancy ratio. When it increases, the proportion of firms that chooses the coordinated market first increases (until u/v is arbitrarily close to one), and then it decreases.

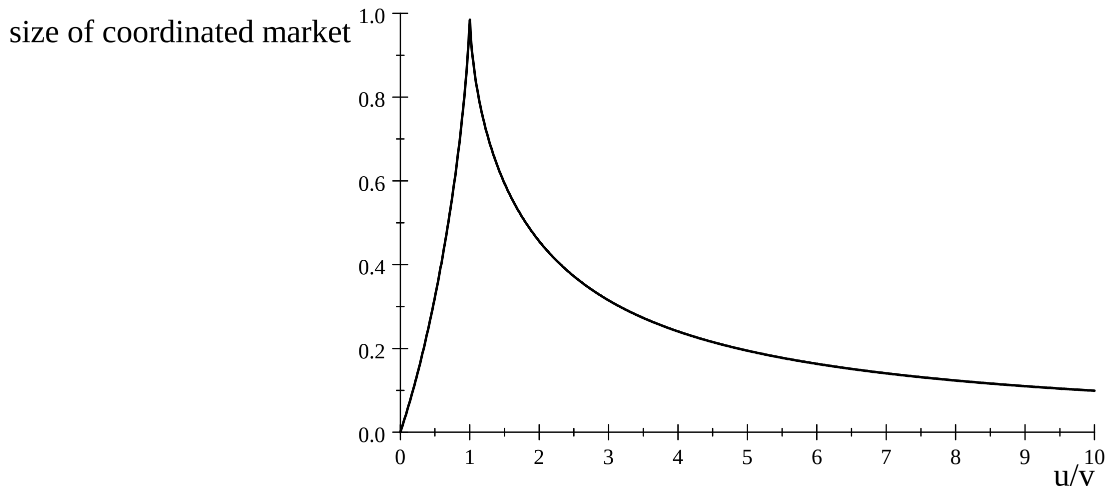

Figure 2: Equilibrium market structure as function of unemployment-vacancy ratio.

2.1 Welfare Analysis I

We note that $dV/d\theta > 0$ and $dU/d\theta < 0$ by using (4) and the results for the equilibrium market structure. The probability α for firms making offers in the coordinated market does not affect the market structure. It can be seen from (8) and indifference condition $V_c = V_s$ that α does not affect firms' utility. Equation (6) tells that worker's utility is larger the smaller is firms' probability of making offers.

Let us next compare the outcomes of the economy with the intermediary to outcomes of pure search economy - an economy which has only a search market with wage posting. Let the value of θ be the same in the two cases. Subscript p denotes the pure search economy, and s and c denote the search submarket and coordinated submarket, respectively, in the economy with an intermediary. The absence of a subscript denotes the economy with an intermediary, comprising of the two submarkets.

We measure the efficiency of the economy by the number of matches.

Proposition 2 *Introducing a middleman increases the number of matches.*

Proof. In the coordinated submarket the number of matches is $M_c = \rho v$, in the search submarket $M_s = (1-\rho)\, v\,(1-e^{-\sigma})$, and in the pure search economy $M_p = v\left(1-e^{-\theta}\right)$. Then $M_c + M_s - M_p = v\left[e^{-\theta} - (1-\rho)\, e^{-\sigma}\right]$. If $\theta > 1$, then $M_c + M_s - M_p > 0$ because $\sigma > \theta$. If $\theta < 1$, using (4) and we obtain $\partial\left(e^{-\theta} - (1-\rho)\, e^{-\sigma}\right)/\partial\rho = \sigma e^{-\sigma} > 0$. This says that putting up a coordinated market increases the number of matches, and therefore $M_c + M_s > M_p$. ■

Denote the workers' and firms' utilities in the economy with an intermediary by U and V, respectively, and denote the utilities in a pure search economy by U_p and V_p, where $U_p = e^{-\theta}$ and $V_p = 1 - e^{-\theta} - \theta e^{-\theta}$.

Proposition 3 *(i) $V > V_p$ if and only if $\theta > 1$ because then $\sigma > \theta$. Otherwise $V < V_p$. (ii) $U > U_p$ if $\theta < 1$, and $U < U_p$ if $\theta \geq 2$. If $1 \leq \theta < 2$, then $U < U_p$ if $\alpha = 1$, and the sign of $U - U_p$ is ambiguous if $\alpha < 1$.*

Proof. (i) $V > V_p$ if and only if $\sigma > \theta$, which holds if and only if $\theta > 1$. Otherwise $V < V_p$. (ii) By (6), $U - U_p = \left(\frac{\rho}{\theta}\sigma(1-\alpha) + 1\right) e^{-\sigma} - e^{-\theta} > 0$ if $\theta < 1$ because $\sigma < \theta$. If $\theta > 1$, then using (4) we have $U > U_p$ if $\alpha < 1 - \frac{(\sigma-1)\theta\left(e^{-\theta} - e^{-\sigma}\right)}{(\sigma-\theta)\sigma e^{-\sigma}}$. If $1 - \frac{(\sigma-1)\theta\left(e^{-\theta} - e^{-\sigma}\right)}{(\sigma-\theta)\sigma e^{-\sigma}} < 0$, then $U < U_p$. Along $1 - \frac{(\sigma-1)\theta\left(e^{-\theta} - e^{-\sigma}\right)}{(\sigma-\theta)\sigma e^{-\sigma}} = 0$, σ is decreasing in θ and larger than θ if $\theta < 2$, and σ is increasing in θ and smaller than θ if $\theta > 2$. Expression $1 - \frac{(\sigma-1)\theta\left(e^{-\theta} - e^{-\sigma}\right)}{(\sigma-\theta)\sigma e^{-\sigma}}$ is negative if σ is larger than it is on $1 - \frac{(\sigma-1)\theta\left(e^{-\theta} - e^{-\sigma}\right)}{(\sigma-\theta)\sigma e^{-\sigma}} = 0$. When $\theta > 2$, then $\sigma > \theta$. These things imply that when $\theta > 2$, then $1 - \frac{(\sigma-1)\theta\left(e^{-\theta} - e^{-\sigma}\right)}{(\sigma-\theta)\sigma e^{-\sigma}} < 0$, and therefore $U < U_p$. ■

Firms prefer an economy with an intermediary to the pure search economy if $\theta > 1$, and vice versa if $\theta < 1$. The reason is that if $\theta > 1$, the job seeker - vacancy ratio in the search market is larger than in a pure search economy. Introducing an intermediary tightens the search market for workers, and it is just the market tightness in the search market which determines the utilities of firms. Workers benefit from the intermediary if $\theta < 1$ because the search market is less tight for them than it is in the pure search economy. If a worker's probability to propose a partition of output increases, this difference increases further. If $\theta > 2$, workers are hurt by the introduction of an intermediary whatever their probability to propose in the coordinated market, because the search market becomes too tight for them. If $1 \leq \theta < 2$, the probability to make an offer in the coordinated market is weighted against a tighter search market, and the sign of $U - U_p$ is ambiguous if $\alpha < 1$.

3 Monopolistic Intermediary and Entry of Firms

In many labor market matching models, it is a standard to assume that the number of vacancies is endogenously determined by free entry and exit. In this section we assume that vacancies can enter the market at an exogenous cost $k > 0$. We assume that u is constant. Then θ is endogenous. The agents make their choices in several stages as in Section 2 except that immediately after the intermediary has announced f, firms enter the market and pay k, and they decide whether to go to the coordinated market or to the search market.

Entry and the choice between two submarkets ensure that a firm's value is equal to zero in both submarkets. Applying (3) and (5), the vacancies' zero profit conditions are

$$V_s = -k + 1 - e^{-\sigma} - \sigma e^{-\sigma} = 0, \tag{8}$$

$$V_c = -k - f + \alpha\left(1 - e^{-\sigma}\right) + (1-\alpha)\left(1 - e^{-\sigma} - \sigma e^{-\sigma}\right) = 0. \tag{9}$$

Firms' entry cost determines the value of σ, the unemployment-vacancy ratio in the search market. The larger k the less there are firms and the larger is σ.

The intermediary's profit maximization problem is given by (7) where he takes θ as given. The equilibrium values of θ and ρ are given by the intersection of $\rho(\theta)$, given by Proposition 1, and $\frac{\theta-\rho}{1-\rho}=\sigma$.

Proposition 4 *The equilibrium market structure is* $\rho=\frac{\sigma}{1-\sigma+\sigma^2}$ *where* σ *is determined by* $-k+1-e^{-\sigma}-\sigma e^{-\sigma}=0$.

Proof. Using (4) and the market structure given in Proposition 1 yields $\sigma=\frac{1}{2\rho}\left(1+\rho+\sqrt{1+2\rho-3\rho^2}\right)$, where $1+2\rho-3\rho^2\geq 0\forall\rho\in[0,1]$. This yields the result. ■

The market structure is non-monotone in σ and thus non-monotone in k: If $\sigma<1$, which by (8) is true if $k<1-2e^{-1}$, then ρ is increasing in σ and thus increasing in k. If $\sigma>1$, which is true if $k>1-2e^{-1}$, then ρ is decreasing in σ and thus decreasing in k.

Proposition 5 *The overall unemployment-vacancy ratio is* $\theta=\frac{2\sigma-2\sigma^2+\sigma^3}{1-\sigma+\sigma^2}$ *where* σ *is determined by* $-k+1-e^{-\sigma}-\sigma e^{-\sigma}=0$.

Proof. Use (4) and Proposition 4. ■

3.1 Welfare Analysis II

By free entry, a firm's expected value is zero. Workers' utility is, using (6) and Propositions 4 and 5, equal to $U=\left(\frac{(1-\alpha)\sigma^2}{2\sigma-2\sigma^2+\sigma^3}+1\right)e^{-\sigma}$, which is decreasing in σ. Because σ is increasing in k by (8), U is decreasing in k. The higher is k, the less firms flow to the economy, and the market is tighter for workers. An increase in firms' probability of making offers in the coordinated market decreases U.

The appropriate measure of efficiency is now the number of matches minus the total entry costs. The total net production is $Q=M_c+M_s-vk=\rho v+(1-\rho)v(1-e^{-\sigma})-vk$.

Proposition 6 *If there is free entry of firms, the economy with an intermediary is more efficient than the pure search economy.*

Proof. Using Propositions 4 and 5 we end up with $Q=\frac{(2\sigma-\sigma+\sigma^2)e^{-\sigma}}{2-2\sigma+\sigma^2}u$. In a pure search economy the unemployment-vacancy ratio ψ is determined by $V_p=-k+1-e^{-\psi}-\psi e^{-\psi}=0$. But this and (8) imply that $\sigma=\psi$. The total net production equals $Q_p=v(1-e^{-\sigma})-vk=u(1-e^{-\sigma}-k)/\sigma$. Using (8) we obtain $Q_p=e^{-\sigma}u$. The relative efficiency from introducing an intermediary is $\frac{Q}{Q_p}=\frac{2\sigma-\sigma+\sigma^2}{2-2\sigma+\sigma^2}>1$ for all $\sigma>0$. ■

Comparing the expected utilities to those in an pure search economy is very simple:

Proposition 7 *(i) $V = V_p = 0$ by entry. (ii) $U = U_p$ if $\alpha = 1$, and $U > U_p$ if $\alpha < 1$.*

Proof. (i) Straightforward, (ii) $U_p = e^{-\psi}$ where ψ is determined by $V_p = -k + 1 - e^{-\psi} - \psi e^{-\psi} = 0$, and $U - U_p = \left(\frac{\rho}{\theta}\sigma(1-\alpha) + 1\right) e^{-\sigma} - e^{-\psi}$. But $V_p = 0$ and (8) imply $\sigma = \psi$. Then $U - U_p = \left(\frac{\rho}{\theta}\psi(1-\alpha) + 1\right) e^{-\psi} - e^{-\psi}$ and the result follows. ■

Workers fare better in the economy with an intermediary than in a pure search economy. In the coordinated submarket they have a chance to propose a division of the output, and if they fail to be hired there, the search submarket provides them the same expected utility as the pure search economy.

4 Bertrand Competition Between Intermediaries

In the above models where the intermediary has monopoly power, it chooses the fee to maximize profit. This is equivalent to choosing the fraction of firms which participates in the coordinated market. In this section we study competition between intermediaries. We assume that they compete with prices, and this leads to zero profits.

We consider an economy where the numbers of firms and workers are fixed, and where there is a "small" number of intermediaries. The order of events is the same as in the monopolistic case. If a firm goes to the coordinated market, it is assumed to observe all intermediaries' fees, and it chooses intermediary i by the fee f_i it announces. In equilibrium all intermediaries charge the same fee. As the number of intermediaries is "small" and the numbers of firms and workers are "large", each intermediary receives the same number of firms and the same number of workers. We also assume that each intermediary suffers cost c from each worker-firm pair it matches. Bertrand competition leads to zero profits, therefore $f = c$ in equilibrium.

Proposition 8 *The both submarkets exist if (i) $c/\alpha \leq e^{-1}$ and (ii) $\theta \in (\sigma_1, \sigma_2)$ where σ_1 and σ_2 are solutions to $\sigma e^{-\sigma} = c/\alpha$. If the both submarkets exist, the equilibrium market structure is determined by $\rho_1 = \dfrac{\theta - \sigma_1}{1 - \sigma_1}$ if $\theta < 1$, and $\rho_2 = \dfrac{\theta - \sigma_2}{1 - \sigma_2}$ if $\theta > 1$.*

Proof. Firms' indifference condition $V_c = V_s$ implies $f = \alpha\sigma e^{-\sigma}$, and intermediaries' profits are zero if $f = c$. This yields $\sigma e^{-\sigma} = c/\alpha$, which has a solution only if $c/\alpha \leq e^{-1}$ (which is the maximum of $\sigma e^{-\sigma}$). If $c/\alpha < e^{-1}$, then $\sigma e^{-\sigma} = c/\alpha$ has two solutions such that $\sigma_1 < 1 < \sigma_2$. Equation (4) gives $\rho = \dfrac{\theta - \sigma}{1 - \sigma}$, and $\rho > 0$ only if $\sigma_1 < \theta < \sigma_2$. ■

The intermediary's cost restricts the range of θ where both coordinated market and search market exist. If the cost is large compared to firms' probability of making an offer in the coordinated market, firms do not participate in that market. Firms' participation in the coordinated market increases in θ if $\theta < 1$, because $\sigma_1 < 1$; it decreases in θ if $\theta > 1$, because $\sigma_2 > 1$.

4.1 The Effects of c and α on Market Structure

The equilibrium market structure depends on the value of c/α:

Proposition 9 *If the two markets exist, the fraction of firms which participates in the coordinated market decreases in c/α.*

Proof. *First,* $\rho = \dfrac{\theta-\sigma}{1-\sigma}$ *implies* $\dfrac{d\rho}{d\sigma} = \dfrac{\theta-1}{(1-\sigma)^2} > (<) 0$ *if* $\theta > (<) 1$. *If* $\theta < 1$, *then* $\sigma_1 < \theta$, *and if* $\theta > 1$, *then* $\sigma_2 > \theta$. *Using* $\dfrac{d\sigma_1}{d(c/a)} > 0$ *and* $\dfrac{d\sigma_2}{d(c/a)} < 0$ *gives* $\dfrac{d\rho}{d(c/\alpha)} = \dfrac{d\rho}{d\sigma}\dfrac{d\sigma}{d(c/\alpha)} < 0$. ■

4.2 Welfare Analysis III

Let us first study how the overall unemployment-vacancy ratio θ affects the firms' and workers' utilities. A firm's value is equal to $1 - e^{-\sigma} - \sigma e^{-\sigma}$ where σ solves $\sigma e^{-\sigma} = c/\alpha$. This implies that θ does not affect firms' value whenever $\theta \in (\sigma_1, \sigma_2)$. If θ is outside these limits, the coordinated market does not exist, and then $\partial V/\partial\theta > 0$.

A workers utility is, using (6) and $\dfrac{\theta-\sigma}{1-\sigma}$, equal to $U = \left(\dfrac{(\theta-\sigma)\sigma(1-\alpha)}{\theta(1-\sigma)} + 1\right) e^{-\sigma}$, and $\dfrac{dU}{d\theta} = \dfrac{\sigma^2(1-\alpha)e^{-\sigma}}{\theta^2(1-\sigma)} > (<) 0$ if $\sigma < (>) 1$. This, in turn, implies that $dU/d\theta > 0$ if $\theta < 1$, and $dU/d\theta < 0$ if $\theta > 1$. The factor determining the change in utility is how ρ/θ behaves as θ changes. If $\theta < 1$, then ρ increases with θ in such an amount that $d(\rho/\theta)/d\theta > 0$, and $dU/d\theta > 0$. If the unemployment-vacancy ratio increases, firms increase their participation in the coordinated market so much that workers fare better. If $\theta > 1$, ρ and θ change into the same direction, and an increase in θ makes workers worse off.

The intermediary's cost and the probabilities of making an offer in the coordinated market affect the market structure. Therefore we expect that c and α affect the utilities of firms and workers, as well as the number of matches and the net production. If c/α increases, σ_1 increases and σ_2 decreases. A firm's value increases in σ, and we conclude that $dV/d(c/\alpha) < 0$ if $\theta > 1$, and $dV/d(c/\alpha) > 0$ if $\theta < 1$. The latter result is first counterintuitive. However, if $\theta < 1$, an increase in c/α increases σ which is the unemployment-vacancy ratio in the search submarket, and this is enough for making firms better off.

If the intermediary's cost increases, σ_1 increases and σ_2 decreases. An increase in σ decreases a worker's expected value in the search market, and as the search market functions as an outside option in the coordinated market, it is clear that U decreases if σ increases. We can conclude that $dU/dc < 0$ if $\theta < 1$, and $dU/dc > 0$ if $\theta > 1$. If α increases, σ_1 decreases and σ_2 increases. Then a worker's expected value in the search market increases in α if $\theta < 1$, and it decreases in α if $\theta > 1$. The "direct" effect of a smaller probability for a worker of making an offer in the coordinated market has a negative impact on U. This implies that if $\theta > 1$, both the direct effect of larger α in the coordinated market and the indirect effect in the search market work in the same direction, decreasing U. But if $\theta < 1$, the effects pull U into opposite directions, and the sign of $dU/d\alpha$ is ambiguous.

Let us next look at how α and c affect the number of matches and total net production. By Proposition 9, it is clear that an increase in c/α results in less matches, because less firms participate in the coordinated market. The net production per worker is $\hat{Q} = \frac{1}{\theta}\left(\rho\left(1-c\right)+\left(1-\rho\right)\left(1-e^{-\sigma}\right)\right) = \frac{1}{\theta}\left(1-\left(1-\frac{\theta-\sigma}{1-\sigma}\right)e^{-\sigma}-\frac{\theta-\sigma}{1-\sigma}c\right)$. Differentiating this with respect to c equals $\frac{d\hat{Q}}{dc} = \frac{\left(\theta-1\right)\left(\sigma e^{-\sigma}-c\right)}{\theta\left(\sigma-1\right)^2}\frac{d\sigma}{dc} - \frac{\rho}{\theta} < 0$ because $\frac{d\sigma}{dc} > (<)0$ if $\theta < (>)1$, and $\sigma e^{-\sigma} \geq c$ if the intermediary makes a nonnegative profit.

Differentiating $\hat{Q}$ with respect to α results in $\frac{d\hat{Q}}{d\alpha} = \frac{\left(\theta-1\right)\left(\sigma e^{-\sigma}-c\right)}{\theta\left(\sigma-1\right)^2}\frac{d\sigma}{d\alpha} > 0$ because $d\sigma/d\alpha < (>)0$ if $\theta < (>)1$, and $\sigma e^{-\sigma} \geq c$ if the intermediary makes a nonnegative profit. If firms' probability of making an offer in the coordinated market increases, more firms choose this market, and this leads to more matches. Having more matches outweighs the total increase in intermediaries' costs.

It is not self-evident that an economy with an intermediary produces more net output than a pure search economy because the intermediary has a cost from arranging matches. If the cost is very high, having more matches can be outweighted by the costs. The difference between the net outputs per worker in the two economies is $\hat{Q} - \hat{Q}_p = \frac{1}{u}\left(\rho v(1-c)+\left(1-\rho\right)v\left(1-e^{-\sigma}\right)\right) - \frac{v}{u}(1-e^{-\theta})$

$= \frac{1}{\theta}\left(\left(\rho(1-c)+\left(1-\rho\right)\left(1-e^{-\sigma}\right)\right)-\left(1-e^{-\theta}\right)\right)$. Then, $\hat{Q} > \hat{Q}_p$ if

$c < \frac{1}{\rho}\left(e^{-\theta}-\left(1-\rho\right)e^{-\sigma}\right)$.

Proposition 10 *$\hat{Q} > \hat{Q}_p$ if $c < e^{-\theta}$.*

Proof. If $\theta > 1$, then $\sigma > \theta$, $e^{-\sigma} < e^{-\theta}$, and $\frac{1}{\rho}\left(e^{-\theta}-\left(1-\rho\right)e^{-\sigma}\right) > 0$. If $\theta < 1$, then $\sigma < \theta$, but $e^{-\theta}-\left(1-\rho\right)e^{-\sigma}$ is positive because $\rho < \theta$. Using (4), $\frac{1}{\rho}\left(e^{-\theta}-\left(1-\rho\right)e^{-\sigma}\right) = \frac{1}{\rho}\left(e^{-\theta}-\left(1-\rho\right)e^{-\frac{\theta-\rho}{1-\rho}}\right)$. The derivative of this with respect to ρ has the same sign as $e^{-\sigma}\left(1-\rho\left(1-\sigma\right)-e^{-\rho(1-\sigma)}\right)$. The latter expression is negative for all $\rho\left(1-\sigma\right) \neq 0$. Then $\frac{1}{\rho}\left(e^{-\theta}-\left(1-\rho\right)e^{-\sigma}\right)$ is decreasing in ρ. It is smallest when $\rho = 1$, then $\frac{1}{\rho}\left(e^{-\theta}-\left(1-\rho\right)e^{-\sigma}\right) = e^{-\theta}$, and the result follows. ■

In a pure search economy, $e^{-\theta}$ is the probability that a firm receives no applicants. As each firm-worker pair produces a unit output, $e^{-\theta}$ is the probability per firm that a unit output is lost. In equilibrium an intermediary provides an immediate match for each firm in the coordinate market. Whenever an intermediary's cost is less then the expected value of output lost, the intermediary institution is beneficial.

As to the comparison of firms' and workers' utilities to those in a pure search economy, Proposition 3 states the results.

5 Conclusion

This article solves the equilibrium market structure in a labor market where agents can meet in a coordinated market or in a search market. We studied the extreme cases of competition between intermediaries: a monopoly and perfect competition. In the monopoly case we showed that the relative size of the coordinated market depends on the unemployment-vacancy ratio, being first increasing in the ratio and then decreasing. The unemployed-vacancy ratio is then endogenized by entry of firms, and the relative size of the coordinated market is first increasing and then decreasing in firms' entry costs. In both cases, introducing an intermediary increases the total output, but the effects on firms' and workers' utilities depend on the unemployment-vacancy ratio. If the number of firms is fixed, the short side of the market generally benefits and the long side of the market generally loses. With free entry, firms are indifferent between having an intermediary or not, but workers gain if they have a positive probability to make an offer in the coordinated market.

Perfect competition among intermediaries implies that a coordinated market exists only if the overall unemployment ratio is within certain limits. The fraction of firms which participates in the coordinated market is determined by the ratio of intermediaries' cost and firms' probability to make an offer in the coordinated market. Finally, the efficiency gain from having intermediaries depends on their unit cost.

We did not analyze a case where there is free entry of firms and perfect competition among intermediaries. This is because there would be two independent conditions for the unemployment-vacancy ratio in the search submarket: one given by the firms' free entry, and another one given by firms' indifference between submarkets and the intermediaries' zero profit. These two conditions hold simultaneously only by chance.

6 Appendix

A1 Second-order condition for the middleman's problem when $\theta > 1$ and the number of firms is fixed

The first-order condition for maximizing profit is $\frac{d\pi}{d\rho} =$

$$e^{-\sigma}\left(\frac{2\rho-\theta}{\rho-1}-\frac{2\rho\left(\rho-\theta\right)}{\left(\rho-1\right)^{2}}+\frac{\rho\left(\rho-\theta\right)^{2}}{\left(\rho-1\right)^{3}}\right) = e^{-\sigma}\left(\frac{\rho^{3}+2\rho-2\rho^{2}+\theta^{2}\rho-\theta\rho^{2}-\theta}{\left(\rho-1\right)^{3}}\right) =$$

0. Let $f\left(\rho\right) = \rho^{3}+2\rho-2\rho^{2}+\theta^{2}\rho-\theta\rho^{2}-\theta$ and $g\left(\rho\right) = \left(\rho-1\right)^{3}$. Then $\frac{d^{2}\pi}{d\rho^{2}} =$ $e^{-\sigma}\left(\frac{g\left(\rho\right)f'\left(\rho\right)-f\left(\rho\right)g'\left(\rho\right)}{g\left(\rho\right)^{2}}\right)+\left(\frac{f\left(\rho\right)}{g\left(\rho\right)}\right)\frac{\partial e^{-\sigma}}{\partial\rho}$ where $f\left(\rho\right) = 0$ when $\frac{d\pi}{d\rho} = 0$. We have $\frac{d^{2}\pi}{d\rho^{2}} = e^{-\sigma}\frac{f'\left(\rho\right)}{g\left(\rho\right)}$, where $f'\left(\rho\right) = 3\rho^{2}+2-4\rho+\theta^{2}-2\theta\rho$ and $g\left(\rho\right) < 0$. Then

$\frac{d^2\pi}{d\rho^2} < 0$ if $f'(\rho) > 0$ at $\frac{d\pi}{d\rho} = 0$.

Use FOC which gives $\theta = \frac{1}{2\rho}\left(1+\rho^2+(1-\rho)\sqrt{1+2\rho-3\rho^2}\right)$. Then $f'(\rho) = \left\{\begin{array}{c} 3\rho^2+2-4\rho+\frac{1}{2\rho}\left(1+\rho^2+(1-\rho)\sqrt{1+2\rho-3\rho^2}\right) \\ \times\left(\frac{1}{2\rho}\left(1+\rho^2+(1-\rho)\sqrt{1+2\rho-3\rho^2}\right)-2\rho\right) \end{array}\right\}$. This implies that $f'(\rho) > 0$ at all $\rho < 1$, and $f'(\rho) = 0$ at $\rho = 1$. This results in $\frac{d^2\pi}{d\rho^2} < 0$ at all $\rho < 1$, and $\frac{d^2\pi}{d\rho^2} = 0$ at $\rho = 1$.The value of ρ that satisfies FOC satisfies also SOC, therefore ρ maximizes intermediary's profit. Because $\rho(\theta)$ is continuous and continuously differentiable for $\theta > 1$, the value of ρ which gives a local maximum for π gives also global maximum.

A2 Second-order condition for the middleman's problem when $\theta < 1$ and the number of firms is fixed

This goes similarly as when $\theta > 1$ except that $\theta = \frac{1}{2\rho}\left(1+\rho^2-(1-\rho)\sqrt{1+2\rho-3\rho^2}\right)$ which gives

$f'(\rho) = \left\{\begin{array}{c} 3\rho^2+2-4\rho+\frac{1}{2\rho}\left(1+\rho^2-(1-\rho)\sqrt{1+2\rho-3\rho^2}\right) \\ \times\left(\frac{1}{2\rho}\left(1+\rho^2-(1-\rho)\sqrt{1+2\rho-3\rho^2}\right)-2\rho\right) \end{array}\right\}$. We have $f'(\rho) > 0$ at all $\rho < 1$ and $f'(\rho) = 0$ at $\rho = 1$. This results in $\frac{d^2\pi}{d\rho^2} < 0$ at all $\rho < 1$, and $\frac{d^2\pi}{d\rho^2} = 0$ at $\rho = 1$. The value of ρ that satisfies FOC satisfies also SOC, therefore ρ maximizes intermediary's profit. Because $\rho(\theta)$ is continuous and continuously differentiable for $\theta < 1$, the value of ρ which gives a local maximum for π gives also global maximum.

References

Biglaiser, G. (1993) "Middlemen as Experts" *RAND Journal of Economics* 24, 212-223.

Bloch, F. and H. Ryder (2000) "Two-Sided Search, Marriages, and Matchmakers" *International Economic Review* 41, No. 1, 93-115.

Gehrig, T. (1993) "Intermediation in Search Market" *Journal of Economics and Management Strategy* 2, No. 1, 97-120.

Halko, M.-L., Kultti, K., and J. Virrankoski (2008) "Search Direction and Wage Dispersion" *International Economic Review* 49, No. 1, 111-134.

Kultti, K. (1999) "Equivalence of Auctions and Posted Prices" *Games and Economic Behavior* 27, 106-113

Miao, J. (2006) "Search Model of Centralized and Decentralized Trade" *Review of Economic Dynamics* 9, 68-92.

Rubinstein, A. and A. Wolinsky (1987) "Middlemen" *Quarterly Journal of Economics* 102, 581-93.

Shevchenko, A. (2004) "Middlemen" *International Economic Review* 45, No. 1, 1-24.

Spulber, D.F. (1996a) "Market Microstructure and Information" *Journal of Economic Perspectives* 10, 135-152.

Spulber, D. F. (1996b) "Market Making by Price-Setting Firms" *Review of Economic Studies* 63, 559-580.

Yavas, A. (1994) "Middlemen in Bilateral Search Markets" *Journal of Labor Economics* 12, No. 3, 406-429.

Yavas, A (1996) "Search and Trading in Intermediated Markets" *Journal of Economics and Management Strategy* 5, No. 2, 195-216.

Aboa Centre for Economics (ACE) was founded in 1998 by the departments of economics at the Turku School of Economics, Åbo Akademi University and University of Turku. The aim of the Centre is to coordinate research and education related to economics in the three universities.

Contact information: Aboa Centre for Economics, Taloustieteen laitos, Assistentinkatu 7, FI-20014 Turun yliopisto, Finland.

Aboa Centre for Economics (ACE) on Turun kolmen yliopiston vuonna 1998 perustama yhteistyöelin. Sen osapuolet ovat Turun kauppakorkeakoulun kansantaloustieteen oppiaine, Åbo Akademin nationalekonomi-oppiaine ja Turun yliopiston taloustieteen laitos. ACEn toiminta-ajatuksena on koordinoida kansantaloustieteen tutkimusta ja opetusta Turun kolmessa yliopistossa.

Yhteystiedot: Aboa Centre for Economics, Taloustieteen laitos, Assistentinkatu 7, 20014 Turun yliopisto.

www.ace-economics.fi

ISSN 1796-3133