Haruo Imai and Hannu Salonen

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ABSTRACT

We study a Baron-Ferejohn (1989) type of bargaining model to which we append an investment stage. As long as no agreement is reached, a new proposer is selected randomly from the player set. A proposal is accepted if at least q players accept it. Prior to the bargaining stage, players may make investments to increase their recognition probabilities in the bargaining game. The investment stage is modeled in the standard way, first suggested by Tullock (1980). When investment costs are the same for all players, no symmetric stationary subgame perfect equilibria in pure investment strategies may exist if unanimity is not needed to reach an agreement. An asymmetric pure stationary equilibrium in a symmetric three-person game exists however when the discount factor is sufficiently high. An equilibrium with symmetric mixed investment strategies exists although payoff functions are not everywhere continuous with respect to investments.

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Contact information

Hannu Salonen (corresponding author)
Department of Economics and PCRC
University of Turku
FI-20014, Finland
Email: hansal (at) utu.fi

Haruo Imai
Kyoto Institute of Economic Research
Kyoto University
Kyoto, Japan Email: imai (at) kier.kyoto-u.ac.jp.

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1. Introduction

We study a modified Baron-Ferejohn type of n -person bargaining model to which we append an investment stage (Baron and Ferejohn 1989). As long as no agreement is reached, a new proposer is selected randomly from the player set, according to the recognition probabilities of the players. A proposal is accepted if at least q players accept it. Prior to the bargaining stage, players may make investments to increase their recognition probabilities in the bargaining game. The investment stage is modeled in the standard way, first suggested by Tullock (1980). The recognition probability of a player is simply the fraction between his investments and the sum of all investments. Players can have different investment costs. We study the existence of stationary subgame perfect equilibria (*SSPE*), and how the equilibrium depends on the decision rule of the bargaining game.

When unanimity is required in the bargaining stage (*i.e.* $q = n$), there is a unique *SSPE*, when investment costs exhibit constant returns to scale. Hillman and Riley (1989) analyzed first the case when players have the same costs. Stein (2002) solved explicitly investment levels of a pure strategy equilibrium. Matros (2006) proved that this equilibrium is unique. Szidarovszky and Okuguchi (1997) have shown uniqueness of equilibrium when players have strictly convex investment costs.

We solve explicitly the equilibrium payoffs, when a q -majority is required in the bargaining stage. We utilize the results of Eraslan (2002) who shows the while there may be several *SSPE* in the Baron-Ferejohn model, the equilibrium payoffs of player i are the same in all equilibria, for all players i. Eraslan doesn't give equilibrium payoffs in explicit form.

After that we study the existence of an *SSPE* in pure strategies. When investment costs are the same for all players, no symmetric stationary subgame perfect equilibria with pure investment strategies may exist if unanimity is not needed to reach an agreement. This has been discovered independently also by Querou and Soubeyran (2011). We show that an equilibrium with symmetric mixed investment strategies exists always despite that in our model payoffs have discontinuities.

The paper is organized in the following way. The notation is given in Section 2 and there we also review some known results about equilibria when unanimity is needed in the bargaining game. In Section 3 we study the q -majority case. We show that no symmetric equilibrium exists with pure investment strategies. An asymmetric equilibrium with pure investment strategies is shown to exist in a three person game with a simple majority rule. A symmetric equilibrium with mixed investment strategies exists when players have the same investment cost.

2. The unanimity model

We analyze a two-stage game such that players make first investments to improve their position in the bargaining game that is played in the second stage. Before bargaining starts players the first period investments become common knowledge. This is called the unanimity model since in the bargaining game any agreement made must be unanimously accepted. We seek for subgame perfect equilibria and study the properties of such equilibria.

2.1. The bargaining game

There are n players indexed by $i \in N = \{1, \ldots, n\}$ who can divide the cake of size 1 if an agreement $x = (x_1, \ldots, x_n)$ is reached how to divide the cake. Player i's utility from an agreement x is simply $x_i, i = 1, \ldots, n$, the size of his share. If no agreement is reached, each player gets utility 0. Agreements x must satisfy $x_i \geq 0$ (individual rationality) and $\sum_i x_i = 1$ (efficiency). Time periods are indexed by natural numbers $t \in \mathbb{N} = \{0, 1, \ldots\}$.

The game starts in period 0 in which a *proposer* $i \in N$ is selected. Each player i has a probability $p_i \geq 0$ of being selected. The selected player i offers an agreement x^i, and the other players, the *responders*, announce simultaneously and independently Yes or No. If all say Yes, the game ends with an agreement x^i. If at least one player says No, then the game moves to the next period and the proposer is selected randomly with probabilities $p_i, i = 1, \ldots, n$. If no agreement is reached in periods $t = 1, \ldots, T$, then in period $T + 1$ the proposer is again selected randomly with probabilities $p_i, i = 1, \ldots, n$. If an agreement x is reached in period t, then player $i's$ utility is $\delta^t x_i, i \in N$, where δ is the common discount factor, $0 < \delta < 1$.

We assume that at each period t, all past choices of all players are common knowledge. Then if no agreement has been reached before period t, a new subgame starts at t.

A *strategy* $s_i = \{(x^{i,t}, (x^{j,t})_{j \neq i})\}_t$ of player i is a plan that specifies (1) what is his proposal $x^{i,t}$ if he is the proposer in period t, and (2) which offers $x^{j,t}$ he accepts from player $j \neq i$ if he is one of the responders in period t, $t = 0, 1, \ldots$. A strategy is *stationary*, if these proposals do not depend on t, and in this case we don't have to index the choices by t.

A profile $s = (s_1, \ldots, s_n)$ is a Nash equilibrium, if s_i maximizes player i's utility when he believes that that other players j choose s_j. A Nash equilibrium s is *subgame perfect*, if it is a Nash equilibrium in every subgame of a bargaining game. A subgame perfect equilibrium s is stationary if every strategy s_i is stationary.

Here is a well-known existence and uniqueness result for stationary equilibria (the proof is available from the authors at request).

Proposition 1. *Given the discount factor* $\delta, 0 < \delta < 1$*, and recognition probabilities* $p = (p_1, \dots, p_n)$*, there is a unique SSPE, s. In this equilibrium player* i *offers* x^i *where he gets* $x^i_i = 1 - \delta + \delta p_i$ *and player* j *gets* $x^i_j = \delta p_j$*,* $i, j \in N, i \neq j$*. Player* i *accepts any offer not smaller than* δp_i*.*

Remark 1. In equilibrium the proposer's payoff decreases in δ whereas responders' payoffs increase in δ. As δ goes to 1, payoffs converge to recognition probabilities. Player i's payoff increases in his recognition probability p_i independently of his role.

Remark 2. Player i's *ex ante* expected equilibrium payoff is simply his recognition probability: $p_i = p_i[1 - \delta + \delta p_i] + (1 - p_i)\delta p_i$.

2.2. The investment game

Before the bargaining game starts, players may make investments that increase their recognition probabilities. Denote by $e_i \geq 0$ the amount (money, effort *etc.*) invested by player i. We assume that p_i depends on investments $e = (e_1, \dots, e_n)$ in the following simple way:

$$p_i(e) = \frac{e_i}{\sum_{j \in N} e_j}, \; i \in N \tag{1}$$

Any player i has strictly positive probability as being selected the proposer iff his investment e_i is strictly positive. If no one invests, every player has the same probability $1/n$ to be selected as the proposer.

We assume that players' investment costs are linear: investing an amount e_i costs $c_i e_i$, where $0 < c_i < 1$ for all $i \in N$. We may assume w.l.o.g. that $c_1 \leq c_2 \leq \cdots \leq c_n$.

The expected payoff $U_i(e)$ of player from investment choices e in the whole game (investment stage and the bargaining stage) is hence, taking into account *Remark* 2.

$$U_i(e) = p_i(e) - c_i e_i, \; i \in N, \tag{2}$$

where $p_i(e)$ is given in equation (1).

We seek for an *SSPE* of this two-stage game, and assume that investment levels become common knowledge before the bargaining game begins. A strategy for player i is $\sigma_i = (e_i, s_i)$ specifying an investment level e_i and a strategy s_i for the bargaining game that might depend on the investments. The assumption of subgame perfection is already built in the payoffs in (2), since there it is assumed that the (unique) *SSPE* is played in the bargaining stage no matter happened in the investment stage. Denote by $\sigma = (\sigma_1, \dots, \sigma_n)$ a strategy profile in this two stage game. Hillman and Riley

(1989) analyze this game when players have the same cost function. Stein (2002) solves the equilibrium investment levels when costs could be different and Matros (2006) proves that the equilibrium is unique. We collect these results in the following proposition.

Proposition 2. *There exist a unique SSPE σ in the two-stage game. Players $i = 1, \ldots, k$ invest strictly positive amounts and players $i > k$ invest nothing where the value of $k \geq 1$ is given by*

$$\frac{\sum_{j=1}^{k+1} c_j}{k} \leq c_{k+1}.$$

The equilibrium investment level of player $i = 1, \ldots, k$ is given by

$$e_i = \frac{k-1}{\sum_{j=1}^{k} c_j} - \frac{(k-1)^2}{[\sum_{j=1}^{k} c_j]^2} c_i$$

The recognition probability of player $i = 1, \ldots, k$ is

$$p_i(e) = 1 - \frac{(k-1)c_i}{\sum_{j=1}^{k} c_j},$$

and his equilibrium payoff is

$$U_i(e) = \left[1 - \frac{(k-1)c_i}{\sum_{j=1}^{k} c_j}\right]^2 = p_i(e)^2.$$

3. The q -majority model

In this section we analyze the case where a proposal in the bargaining stage is implemented if at least q players accept it, $n/2 < q < n$. Otherwise the model is as before and the existence and properties of stationary subgame perfect equilibria are studied.

Baron and Ferejohn (1989) study explicitly the bargaining game with odd n and $q = (n+1)/2$, *i.e.* simple majority. They find that an *SSPE* is unique when recognition probabilities are the same. They also demonstrate that with unequal recognition probabilities stationary equilibrium need not be unique. Eraslan (2002) shows that even in this case the equilibrium payoffs are unique.

We can utilize these results. Since equilibrium payoffs in the bargaining stage are unique, it suffices to find one stationary subgame perfect equilibrium for the bargaining game, compute the corresponding payoffs and study what are the equilibrium investments.

3.1. The bargaining game

Let $p = (p_1, \dots, p_n)$ be a vector of recognition probabilities, and assume w.l.o.g. that $p_1 \geq \cdots \geq p_n$. Fix the voting rule $q \in \mathbb{N}$, $n/2 < q < n$. It is intuitively clear that equilibrium payoffs at this stage must be increasing in the recognition probabilities (Eraslan 2002, p. 21, shows this formally).

Another property that is quite intuitive is that a minimal winning coalition will form and share the unit cake among its members (Eraslan 2002, p.16). That is, at most q players can get strictly positive payoffs. It may be less obvious that generally this minimal winning coalition is formed randomly: the proposer in each period makes a serious offer only to $q-1$ other players, and usually at least some of these players are chosen randomly.

A player with a low recognition probability has greater chances to be selected in the winning coalition than a player with higher recognition probability (this follows from Theorem 4, p. 20 in Eraslan 2002). This again is intuitive: players with low recognition accept lower offers. We need to prove some additional properties of equilibria that to our knowlegde have not been shown earlier in the literature.

Let $|\{k, \dots, n\}| = q$, so $\{k, \dots, n\}$ is the minimal winning coalition whose members have the lowest recognition probabilities. Note that $k = n - q + 1$, and call player k the critical player.

Given an *SSPE* in this bargaining game, denote by $y_i = x_i^j$ the offer that player j makes to i. Now $y_i = \delta v_i$ must hold in equilibrium, where v_i is the continuation value for player i, and hence y_i does not depend on the identity of the proposer.

Denote by $x_i = x_i^i$ the amount that player i reserves for himself when he makes an offer. Then $x_i = 1 - \sum_{i \in M} y_i$, where M is the subset of players to whom i makes an offer, $|M| = q - 1$.

Let $K = \{j \mid x_j = x_k\}$, so K consists of all those players who reserve to themselves the same amount as the critical player k. Note that it holds that $i, j \in K, i < j$, implies $h \in K$ for all h such that $i \leq h \leq j$.

Let r_i be the *invitation probability*, the probability that player i gets an offer when he himself is not the proposer. Let $H \equiv \{j > k \mid j \notin K\}$, so H consists of those players j who keep to themselves an amount $x_j < x_k$ when they make an offer. Let $L = \{j < k \mid j \notin K\}$, so L consists of those players j who keep to themselves an amount $x_j > x_k$ when they make an offer. (It is possible that L and/or H is empty.) We have the following result.

Lemma 1. *If r_i satisfies $0 < r_i < 1$, then $i \in K$. If $i \in H$, then $r_i = 1$. If $i \in L$, then $r_i = 0$.*

Proof. Note that the cheapest way to form a minimal winning coalition is to use players in $K \cup H$. (Of course any *proposer* outside $K \cup H$ belongs to minimal winning coalition as well.) This follows since $i \in L$ means $x_k < x_i$ and since $p_i > p_k$, i also rejects more offers than k. Hence k is a more reasonable partner in a minimal winning coalition and so $r_i = 0$ for $i \in L$.

If $i \in H$, then $x_i < x_k$. Since $|H| < q$, any least cost minimal winning coalition always contains i as a member. Therefore $r_i = 1$. It follows that $0 < r_j < 1$ implies $j \in K$. □

The continuation value v_i of player $i \in K$ satisfies

$$y_k = \delta[p_i(a + y_k) + (1 - p_i)r_i y_k] = \delta v_i \tag{3}$$

where $a + y_k = x_k$ is the amount player reserves for himself, $a > 0$. It follows of course that $v_i = v_k$ for all $i \in K$ since players in K behave the same way and are treated the same way by the other players.

It is clear that when player k selects his partners into a minimal winning coalition, the players $j = k+1, \ldots, n$ can be included with the minimal cost. This implies that

$$a + y_k + \sum_{i>k} y_i = 1. \tag{4}$$

It follows immediately that when player $i \in H$ makes an offer, the least cost minimal winning coalition for him is $\{k, \ldots, n\} \setminus \{i\}$. Hence by (4) players $i \in H$ get $x_i = a + y_i$ when they make an offer.

For players $i \in L$ we get from (4) that $x_i = x_k = a + y_k$. These players take the place of the critical player k and hence get the same x_k as k. A player $i \in L$ is never invited in a minimal winning coalition. However in the beginning of every period he has probability p_i of becoming the proposer. Therefore his expected equilibrium value v_i satisfies

$$p_i(a + y_k) = v_i \tag{5}$$

The continuation value v_i for player $i \in H$ satisfies

$$y_i = \delta(p_i(a + y_i) + (1 - p_i)y_i) = \delta v_i \tag{6}$$

since $i \in H$ is invited in every minimal winning coalition with probability one. We can solve y_i from (6) as a function of a:

$$y_i = \frac{p_i a \delta}{1 - \delta}. \tag{7}$$

The equilibrium values of a and y_k are given in the proposition below. With these numbers we get the equilibrium offers and value functions from equations (3) - (7).

Proposition 3. *In an SSPE the numbers a and y_k are given by*

$$y_k = \frac{\delta(1-\delta)p(K)}{D} \tag{8}$$

$$a = \frac{(1-\delta)(|K| - \delta(q - |H| - p(L)))}{D} \tag{9}$$

where $p(A) = \sum_{i \in A} p_i$, for each $A \subset \{1, \dots, n\}$ and

$$D = (\delta p(L) + |K|)\big[1 - \delta + \delta p(H)\big] - \delta(q - |H|)\big[(1-\delta)p(L) + p(H)\big].$$

Proof. Note that a minimal winning coalition must contain $q - |H|$ members from K, when the proposer is $i \in K \cup H$. If the proposer is $i \in L$, then the minimal winning coalition contains the proposer i and $q - 1 - |H|$ members from K as well as the set H. It follows that the minimal offers y_i that players in $K \cup H$ are willing to accept must satisfy

$$a + (q - |H|)y_k + \sum_{i \in H} y_i = 1. \tag{10}$$

By (7) this is equivalent to

$$a + (q - |H|)y_k + \frac{a\delta}{1-\delta} \sum_{i \in H} p_i = 1. \tag{11}$$

The value v_i of any player i satisfies

$$p_i(y_i + a) + (1 - p_i)r_i y_i = v_i. \tag{12}$$

Each period a unit cake is divided among the players. The division may be random, but each realization z is a nonnegative n -dimensional vector whose components add up to 1. Players have common beliefs about what is the probability of each such z. Since each v_i is just the expected value of the shares z_i player i gets, it follows that $\sum_i v_i = 1$. By equations (3) and (5) - (7) this is equivalent to

$$p(L)(a + y_k) + \frac{|K|y_k}{\delta} + \frac{ap(H)}{1-\delta} = 1. \tag{13}$$

Equations (11) and (13) form a linear 2×2 linear system with variables a and y_k. The unique solution of this system is given by (8) and (9). $\square$

We can now solve for players' value functions v_i.

Corollary 1. *The value functions in an SSPE are the following:*

$$v_i = \begin{cases} p_i\{|K| - \delta(q - |H| - p(L))\}/D & i \in H \\ (1-\delta)p(K)/D & i \in K \\ p_i(1-\delta)\{|K| - \delta(q - |H| - p(L) - p(K))\}/D & i \in L \end{cases} \tag{14}$$

where D is given by Prop. 6.

Proof. The numbers a and y_k given by (8) and (9) are inserted into the value functions in equations (3), (5) and (6). □

3.2. On the existence of a pure SSPE

Let us first analyze the possibility that all players belong to K in equilibrium. It turns out that this is impossible. Suppose however that such an equilibrium exists. Then every player accepts any offer that is not below y_k and every player keeps to himself the amount $a + y_k$ when he is the proposer. The values of these numbers are given by (8) and (9) when $H = L = \emptyset$ and $K = \{1, \dots, n\}$:

$$y_k = \frac{\delta}{n}, \text{ and } a = 1 - \frac{q\delta}{n}. \tag{15}$$

The expected value from bargaining is $v_k = 1/n$. Since everybody gets the same from the bargaining game, the investments must also be the same. Let us denote by h the equilibrium investment level of any player.

As before let the cost levels satisfy $0 < c_1 \le \cdots \le c_n < 1$. If i has an incentive to invest less than h then naturally also player n has such an incentive. Note that if n invests $e < h$ then either 1) $n \in H$ or 2) $n \in K$ in the new equilibrium of the bargaining game. But if case 2) holds then n will deviate for sure since his expected payoff from bargaining is still $1/n$ by (14) but his investment cost is lower. Hence case 1) must hold and we should show that deviation to $h < e$ is not profitable.

Case 1. The new recognition probabilities are

$$p_n = \frac{e}{(n-1)h + e}, \text{ and } p_i = \frac{h}{(n-1)h + e}, \text{ for } i < n. \tag{16}$$

Inserting the values $H = \{n\}$, $K = \{1, \dots, n-1\}$ in the formula of v_n in equation (14) of Cor. 2, we get that the value from bargaining $\hat{v}_n$ after deviation is

$$\hat{v}_n = \frac{p_n[n - 1 - \delta(q-1)]}{p_n[\delta(n-1) - \delta(q-1)] + (n-1)(1-\delta)}. \tag{17}$$

The value $\hat{v}_k$ for other players $i < n$ is found in the same way:

$$\hat{v}_k = \frac{(1-\delta)(1-p_n)}{p_n[\delta(n-1) - \delta(q-1)] + (n-1)(1-\delta)}. \tag{18}$$

It follows from Eraslan (2002, p. 21) that $H = \{n\}$ implies $\hat{v}_k \geq \hat{v}_n$. By using (17) and (18) this is seen to be equivalent to

$$p_n < \frac{1-\delta}{n-\delta q}. \tag{19}$$

It is straightforward to check that in (19), $(1-\delta)/(n-\delta q) < 1/n$, hence the probability p_n is bound away from $1/n$. But this means that investing only slightly less than h player n will stay in the group K also in the new bargaining equilibrium, and hence case 2) holds after all. We have the following result.

Proposition 4. *When $q < n$, there exists no pure* SSPE *such that $K = \{1, \ldots, n\}$, that is, no such pure equilibria that all players invest the same amount in publicity.*

Independently from us, Querou and Soubeyran (2011) have reached this same result.

However, there might exist *asymmetric* pure strategy equilibria. We demonstrate this for three person games.

Proposition 5. *If $N = \{1, 2, 3\}$, $q = 2$ and $c_i = c > 0$ for every $i \in N$, then let he investment level of player 1 be $h_1 = (2-\delta)/c(3-\delta)^2$, and let the investment levels of players 2 and 3 satisfy $h_2 + h_3 = (2-\delta)/2c(3-\delta)^2$ and $1/8 < h_3/h_2 < 1/5$. Then there exists a $\bar{\delta} < 1$ such that for $\delta > \bar{\delta}$, there exists an SSPE with investment levels $h = (h_1, h_2, h_3)$. The equilibrium recognition probability p_1 of player 1 is $2/3$. The recognition probabilities of players 2 and 3 satisfy $p_2 + p_3 = 1/3$ and $5/18 < p_2 < 8/27$ and $1/27 < p_3 < 1/18$.*

Proof. See the Appendix. □

3.3. On the existence of a symmetric mixed SSPE

Although there are no symmetric *SSPE* with pure investment strategies, there exists a symmetric *SSPE* with mixed investment strategies, when players have equal costs $c > 0$. After we have shown this, we provide a numerical example for a three person majority voting case.

First note that investment levels $e_i > 1/c$ are strictly dominated by zero investment. Hence we need only consider mixed investment strategies over

$[0, 1/c]$. Denote by Δ the compact metric space of all probability measures over $[0, 1/c]$, equipped with the weak convergence topology (Prohorov metric). That is, $p \leq_s q$ iff $\int f(x)dp \leq \int f(x)dq$, for all bounded measurable nondecreasing functions $f : [0, 1] \to \mathbb{R}$. Denote by Δ^n the set of mixed strategy profiles with product topology. We say that profile p is symmetric, if $p_i = p_1$ for all players i.

Let $B_i(q)$ be the set of all mixed best replies of player i against a mixed profile q, and denote by B_i the best reply correspondence of i. The product correspondence is denoted by B. As usual, pure strategies $x \in [0, 1/c]$ are viewed as degenerate mixed strategies, and with a slight abuse of notation we may denote $x \in B_i(q)$ or $x \in \Delta$.

Proposition 6. *If players have the same investment cost c, then there exists an* SSPE *in which all players use the same mixed investment strategy q_1. Moreover, $q_1(\{0\}) = 0$, that is, only strictly positive investment levels get positive probability.*

Proof. See the Appendix. □

APPENDIX

Proof of Proposition 5. The value functions $v_i(h)$ (*gross of investment costs*) from investments $h = (h_1, h_2, h_3)$ are (Proposition 3 and Corollary 1) the following.

$$
\begin{aligned}
v_1(h) &= \frac{(2-\delta)h_1}{(2-\delta)h_1 + 2(h_2+h_3)} \\
v_i(h) &= \frac{h_2+h_3}{(2-\delta)h_1 + 2(h_2+h_3)}, \ i = 2,3.
\end{aligned} \tag{20}
$$

At an interior equilibrium h the following first order condition should be satisfied.

$$
\begin{aligned}
\frac{\partial v_1(y)}{\partial e_1} - c &= \frac{2(2-\delta)(h_2+h_3)}{[(2-\delta)h_1 + 2(h_2+h_3)]^2} - c = 0 \\
\frac{\partial v_i(y)}{\partial e_i} - c &= \frac{h_1(2-\delta)}{[(2-\delta)h_1 + 2(h_2+h_3)]^2} - c = 0, \ i = 2,3.
\end{aligned} \tag{21}
$$

It must be true that $h_1 = 2(h_2+h_3)$, and the investments satisfy:

$$
\begin{aligned}
h_1 &= \frac{2-\delta}{c(3-\delta)^2} \\
h_2 + h_3 &= \frac{2-\delta}{2c(3-\delta)^2}
\end{aligned} \tag{22}
$$

We must show that equations (22) constitute an equilibrium. First, note that there cannot be a profitable deviation for any player such that the equilibrium configuration remains $L = \{1\}$ and $K = \{2,3\}$. Hence for example for player 1, a deviation can be profitable only if it results *e.g.* in a configuration $K = \{1,2,3\}$, or $K = \{2,3\}, H = \{1\}$, and so on. Similarly, player 2 or 3 can have only such profitable deviations that a new configuration emerges. For example $K = \{1,2\}, H = \{3\}$ could in principle result from a deviation of player 2. However, using the value functions in Corollary 1, one can show that none of these possible deviations is profitable in the limit when δ approaches 1. Moreover, in the limit the restrictions for investment levels h_i hold as claimed. We will now complete the proof.

We have several distinct cases depending upon the values of investment level of each player i. Suppose that at least one player invests a positive amount.

Noting that K cannot be empty and $|H|, |L| < 2$, we have cases with $|K| = 3, |H| = 2$ and either $|L| = 1$ or $|H| = 1$, and $|H| = |K| = |L| = 1$. We denote these cases respectively KKK, LKK, KKH, and LKH.

When one of those patterns is the equilibrium configuration, the equilibrium payoffs (net of investment cost) are given by the following (we assume $p_1 \geq p_2 \geq p_3$, to avoid duplication).

	player 1	player 2	player 3
KKK	$1/3$	$1/3$	$1/3$
LKK	$\frac{(2-\delta)p_1}{2-\delta p_1}$	$\frac{(1-p_1)}{2-\delta p_1}$	$\frac{(1-p_1)}{2-\delta p_1}$
KKH	$\frac{(1-\delta)(1-p_3)}{2(1-\delta)+\delta p_3}$	$\frac{(1-\delta)(1-p_3)}{2(1-\delta)+\delta p_3}$	$\frac{(2-\delta)p_3}{2(1-\delta)+\delta p_3}$
LKH	$\frac{p_1(1-\delta)(1-\delta p_3)}{1-\delta+\delta^2 p_1 p_3}$	$\frac{(1-\delta)p_2}{1-\delta+\delta^2 p_1 p_3}$	$\frac{p_3(1-\delta+\delta p_1)}{1-\delta+\delta^2 p_1 p_3}$

Table 1. Equilibrium payoffs

The possibility of each configuration XYZ is determined by the values of the invitation probabilities $r_i \in [0,1]$. The conditions are

$$\textit{the configuration } XYZ = \begin{cases} KKK & \text{if } p_1 \leq \frac{1}{3-\delta}, \text{ and } p_3 \geq \frac{1-\delta}{3-2\delta} \\ LKK & \text{if } p_1 \geq \frac{1}{3-\delta}, \text{ and } p_3 \geq \frac{(1-\delta)(1-p_1)}{2(1-\delta)+\delta p_1} \\ KKH & \text{if } p_3 \leq \min\{\frac{1-\delta}{3-2\delta}, \frac{1-2p_1}{1-\delta p_1}\} \\ LKH & \text{if } \frac{1-2p_1}{1-\delta p_1} \leq p_3 \leq \frac{(1-\delta)(1-p_1)}{2(1-\delta)+\delta p_1} \end{cases}$$

These exhaust all cases and at the boudary of each case, payoffs are equal.

We shall refer to the configuration LKK with $L = \{i\}$ as $iLKK$, and the configuration $LKH = \{i\}\{j\}\{k\}$ as $iLKHk$. We may also refer to KKH with $H = \{i\}$ as $KKHi$.

Given these data, we confirm that the investment levels shown in the proposition (h_1, h_2, h_3) comprise an equilibrium by showing that each investment level is a best response.

Given investments (e_1, e_2, e_3), denote equilibrium payoffs from the bargaining subgame by $V_i(e_1, e_2, e_3)$ for $i = 1, 2, 3$. Resulting net payoff is given by $V_i(e_1, e_2, e_3) - ce_i$.

These subgame payoffs V_1 for player 1 are determined as function of e_1 as follows, given other players' investment levels h_2, h_3:

$$V_1(e_1, h_2, h_3) = \begin{cases} \frac{(2-\delta)e_1}{(2-\delta)e_1+2(h_2+h_3)}, & e_1 \geq \frac{h_2+h_3}{2-\delta}, \ (1LKK) \\ 1/3, & (2-\delta)h_2 - h_3 \leq e_1 \leq \frac{h_2+h_3}{2-\delta}, \ (KKK) \\ \frac{e_1+h_3}{2(e_1+h_3)+(2-\delta)h_2}, & e^0 \leq e_1 \leq (2-\delta)h_2 - h_3, \ (2LKK) \\ \frac{e_1(1-\delta+\delta h_2)}{(1-\delta)(e_1+h_2+h_3)^2+\delta^2 e_1 h_3}, & 0 \leq e_1 \leq e^0, \ (2LKH1) \end{cases}$$

where at e^0 satisfies

$$\frac{e^0}{e^0+h_2+h_3} = \frac{(1-\delta)(1-\frac{h_2}{e^0+h_2+h_3})}{2(1-\delta)+\delta(\frac{h_2}{e0+h_2+h_3})} < \frac{1-\delta}{3-2\delta}$$

and e^0 tends to 0 as δ tends to 1. Also note that given the values in the proposition, for δ sufficiently large, recognition probabilities never enter the regions corresponding to KKH.

Define

$$f_1(e_1; h_2, h_3) = \frac{(2-\delta)e_1}{(2-\delta)e_1 + 2(h_2+h_3)}.$$

Then f_1 is a nonnegative, concave, and increasing function of e_1. Also note that $\frac{df_1}{de_1}(h_1; h_2, h_3) = c$. Similarly define

$$\begin{aligned} f_2(e_1; h_2, h_3) &= \frac{e_1+h_3}{2(e_1+h_3)+(2-\delta)h_2} \\ f_3(e_1; h_2, h_3) &= \frac{e_1(1-\delta+\delta h_2)}{(1-\delta)(e_1+h_2+h_3)^2+\delta^2 e_1 h_3}. \end{aligned}$$

Then f_2 is a positive, concave, and increasing function of e_1 while f_3 is a nonnegative increasing function of e_1. Define

$$g_2(e_1; h_2, h_3) = \max\{f_2(e_1; h_2, h_3), f_2(e^0; h_2, h_3)\}.$$

If we can show that h_1 is the best response for the modified payoff function based on the modified subgame payoff function $V_1^*(e_1, h_2, h_3) = \max\{f_1, \min\{1/3, g_2\}\}$, then h_1 is the best response for the original payoff function. Since e^0 tends to 0, we essentially compare the best payoff in the $2LKK$ region with the equilibrium payoff.

Since f_1 and f_2 are both concave, a sufficient condition is

$$\begin{aligned} \frac{df_2}{de_1}((2-\delta)h_2 - h_3; h_2, h_3) &\le \frac{df_1}{de_1}(h_1; h_2, h_3) = c, \text{ and} \\ f_2((2-\delta)h_2 - h_3; h_2, h_3) - c((2-\delta)h_2 - h_3) &\le f_1(h_1; h_2, h_3) - ch_1. \end{aligned}$$

Now

$$\begin{aligned} \frac{df_2}{de_1}((2-\delta)h_2 - h_3; h_2, h_3) &= \frac{(2-\delta)h_2}{[2(((2-\delta)h_2 - h_3)+h_3)+(2-\delta)h_2]^2} \\ &= \frac{1}{9(2-\delta)h_2}. \end{aligned}$$

For h_2 and h_3 satisfying $1/8 < h_3/h_2$, there is δ sufficiently close to 1 so that $2(3-\delta)^2(h_2+h_3) \le 9(2-\delta)^2 h_2$ holds and hence $\frac{1}{9(2-\delta)h_2} \le c$.

We know that $f_2((2-\delta)h_2 - h_3; h_2, h_3) = 1/3$ and $f_1(h_1; h_2, h_3) = \frac{2-\delta}{3-\delta}$. Thus, given $1/5 > h_3/h_2$, by choosing δ sufficiently close to 1, we have

$$\frac{\frac{2-\delta}{3-\delta} - \frac{1}{3}}{2(h_2+h_3) - [(2-\delta)h_2 - h_3]} = \frac{3-2\delta}{3(3-\delta)(\delta h_2 + 3h_3)} \geq c.$$

Hence h_1 is the best response for player 1.

The subgame payoff V_2 for player 2 is given by the following formula, given h_1, h_3:

$$V_2(h_1, e_2, h_3) = \begin{cases} f_1(e_2; h_1, h_3), & e_2 \geq \frac{h_1+h_3}{2-\delta}, \ (2LKK) \\ 1/3, & (2-\delta)h_1 - h_3 \leq e_2 \leq \frac{h_1+h_3}{2-\delta}, \ (KKK) \\ f_2(e_2; h_1, h_3), & e^{00} \leq e_2 \leq (2-\delta)h_1 - h_3, \ (1LKK) \\ f_3(e_2; h_1, h_3), & 0 \leq e_2 \leq e^{00}, \ (1LKH2) \end{cases}$$

where e^{00} satisfies

$$\frac{e_2}{h_1 + e^{00} + h_3} = \frac{(1-\delta)(1 - \frac{h_1}{e^{00}+h_1+h_3})}{2(1-\delta) + \delta(\frac{h_1}{e^{00}+h1+h_3})} < \frac{1-\delta}{3-2\delta},$$

and e^{00} tends to 0 as δ tends to 1.

Again we can define the modified payoff function V_2^* and confirm that h_2 is the best response for it. A sufficient condition for h_2 being a best response is

$$\frac{df_1}{de_1}\left(\frac{h_1+h_3}{2-\delta}; h_1, h_3\right) \leq c = \frac{df_2}{de_1}\left(h_2; h_1, h_3\right).$$

This holds because

$$c = \frac{(2-\delta)h_1}{[(2-\delta)h_1 + 2(h_2+h_3)]^2} = \frac{2-\delta}{(3-\delta)^2 h_1} > \frac{2(2-\delta)}{9(h_1+h_3)}$$

for δ large enough.

Argument for player 3 is similar to the case of player 2. □

Proof of Proposition 6. Note that $B_i(q)$ is nonempty, convex and compact for every $q \in \Delta^n$ such that $q_j(\{0\}) < 1$ for at least some player $j \neq i$. However, payoff functions are discontinuous at investment levels $e_i = 0, i = 1, \ldots, n$. Therefore, consider first games $G(m)$ such that every players investment must be at least $1/mn, m = 1, \ldots$. Denote the mixed strategies over $[1/mn, 1/c]$ by Δ_m and the set of corresponding profiles by Δ_m^n. Then the best reply correspondences in $G(m)$ are upper hemicontinuous with nonempty convex values. By the Fan-Glicksberg theorem, in the game $G(m)$ there exists an $SSPE$ with mixed investment strategies $q = (q_1, \ldots, q_n)$. We show first (*Step 1.*) that in the game $G(m)$ there also exists a symmetric equilibrium, $q_1 = \cdots = q_n$, and after that we show (*Step 2.*) that this holds also in the original game where also zero investments are allowed.

Step 1. Denote the best reply correspondences in the game $G(m)$ by $B_i^m(p)$. Take a symmetric profile $p \in \Delta_m^n$, and note that $B_1^m(p) = \cdots = B_n^m(p)$. Then the correspondence b_1^m on Δ_m defined by $b_1^m(p_1) = B_1^m(p)$ is an upper hemicontinous correspondence, and b_1 has nonempty convex values. Hence by the Fan-Glicksberg theorem, b_1^m has a fixed point $q_1^m \in \Delta_m$. By construction the profile $q^m = (q_1^m, \ldots, q_1^m)$ is a symmetric profile of mixed investment strategies in an $SSPE$ of the game in which investment levels must be at least $1/mn$.

Step 2. Let m go to infinity. Choose a symmetric mixed equilibrium profile q^m from each game $G(m)$. Then the sequence $\{q^m\}_m$ has a convergent subsequence $\{q^{m_k})\}_k$. Note that the limit of such a sequence is a symmetric profile. There are two possible cases: A) for some convergent subsequence, the limit $q = (q_1, \ldots, q_1)$ is such that $q_1(\{0\}) < 1$; B) for all convergent subsequences, the only limit $q = (q_1, \ldots, q_1)$ is such that $q_1(\{0\}) = 1$.

Case A. By continuity of payoff function at strictly positive investment levels, the limit q is a symmetric equilibrium if $q_1(\{0\}) = 0$. Recall that we did not define payoffs if all players choose zero investments. However, we *could* define that the recognition probability is $1/n$ for all players in such a case. Nevertheless, even then payoffs would be discontinuous at zero investments, and then the Fan-Glicksberg theorem does not apply (this does *not* mean that a mixed profile with an atom at 0 could not be an equilibrium). We will rule out the possibility that $q_1(\{0\}) > 0$.

Assume $q_1(\{0\}) > 0$. We may assume *w.l.o.g.* that the sequence $\{q^m\}_m$ itself converges to q. By convergence, the strategy q_1^m must have an atom at some x_m near $1/nm$ for large values of m, such that $x_m \longrightarrow 0$ (by taking a subsequence of $\{x_m\}_m$ if necessary) and $q_1^m(\{x_m\}) \longrightarrow q_1(\{0\})$. The expected payoff from bargaining (excluding the investment cost) must be $1/n$ for every player in each game $G(m)$ (note that this holds also in the limit, given that zero investments lead to equal recognition probabilities). This is so because unit cake is shared in every game, and investment strategies and payoff functions are symmetric.

Consider a small investment level $\varepsilon > 0$. Suppose that player i deviates in such a way that he puts all the probability mass $q_1([0, \varepsilon])$ on ε, but otherwise the strategy is left unchanged. Then player i's expected payoff from bargaining is almost one, conditional that other players investment is zero and he invests ε, because in such a case his recognition probability is almost one.

The probability of such and event is roughly $q_1(\{0\})^n$ if q_1 has no atoms in $(0, \varepsilon)$, and it is larger than $q_1(\{0\})^n$ if q_1 has an atom in $(0, \varepsilon)$. His investment cost has increased by at most $c\varepsilon$. Hence a deviation can be made profitable by choosing a sufficiently small ε. But then q^m cannot be an equilibrium investment profile in $G(m)$, a contradiction. So the claim holds in Case A.

Case B. If all convergent subsequences of $\{q^m\}_m$ have a limit q such that $q_1(\{0\}) = 1$, then the sequence itself converges to q. The expected payoff from bargaining is $1/n$, and the expected cost of investment goes to zero. Hence the expected equilibrium payoff is almost $1/n$ for every player for large m. By convergence, for each $\varepsilon \in (0, 1)$, and each $\delta \in (0, 1)$, there is M such that $q_1^m([0, \varepsilon]) > 1 - \delta$ when $m > M$. Consider a deviation by player i such that he puts the probability mass $q_1([0, \varepsilon])$ on ε, and otherwise keeps the strategy as it was. By the similar argument that was used in Case A, one can show that such a deviation can be made profitable. Therefore $q(m)$ cannot be an equilibrium investment profile in $G(m)$ for large m. Hence Case B is impossible. $\square$

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Contact information: Aboa Centre for Economics, Department of Economics, Rehtorinpellonkatu 3, FI-20500 Turku, Finland.

www.ace-economics.fi

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