Growth and precautionary saving in the Ramsey-Cass-Koopmans economy

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Abstract

This paper introduces precautionary saving motive into the basic Ramsey-Cass-Koopmans economy. The model is used to explain the positive correlation across countries between saving and growth. This correlation is difficult to reconcile with standard growth model with plausible parameters values, since forward looking agent does not wait higher income to materialize but consumes it today. It is shown that the standard growth model extended by precautionary saving motives can imply that increase in growth can cause increase in saving.

JEL Codes: D91, E21

Keywords: Growth, Saving

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1 Introduction

The positively correlated growth and saving rates across countries are well known empirical fact among economist at least since 1960. However, the interpretation of that correlation is still controversial. On the one hand, this empirical fact is explained by using growth models where higher saving leads to temporary or permanent increase in growth depending on the nature of growth model. On the other hand, recent and growing evidence suggest that causality runs from growth to savings, not the other way around. In that case the positive association between growth rates and saving rates is problematic for standard models, since those models imply that higher growth cause lower saving. This paper shows that when the basic Ramsey-Cass-Koopmans growth model is augmented by precautionary saving motive, can this model generate a positive correlation between growth rate and saving rate even when causality between these two variables runs from growth to savings.

As we mentioned this paper introduces precautionary saving into the basic Ramsey-Cass-Koopmans economy. General equilibrium models solved under uncertainty are usually very complicated, but here we assume that income process of representative household depends on a Poisson process. That is, households' has a constant probability for reduced income level, which generates precautionary saving motive. This is the only source of uncertainty in the model. This assumption simplifies analysis greatly, even so, that under restricted parameter values model is tractable.

Toche (2005) solved this type of model in a partial equilibrium and this paper expands this method to general equilibrium. As noted by Toche and Carrol and Kimball (2006) the exact form of uncertainty does not seems to affect behavior greatly, and hence, by a Poisson process we can generate a realistic precautionary saving behavior. Obviously, behavior differs from the benchmark case where labor income follows i.i.d process.

The goal is to show that precautionary saving motive could be an underlying reason to detected relationship between growth and saving. Hence, the traditional interpretation for the relationship between saving and growth is wrong or overstated as argued by Carroll and Weil (1994).[1] It is shown in the paper that when causality is running from

[1] Still we do not take a view that the correlation is only caused by causality which

growth to savings, can the Ramsey-Cass-Koopmans model still be used to explain a positive association between growth and saving, but the model must be augmented by precautionary saving motive. We are not trying to model the effects of the most realistic uncertainty process in a general equilibrium, but we show that precautionary saving behavior can potentially explain the empirical fact concerning growth and savings.

To summarize, this paper has two contributions: Firstly, we introduce precautionary saving motive into the basic Ramsey-Cass-Koopmans growth model in a way that model can even kept tractable. Secondly, we argue that precautionary saving behavior can be a potential explanation for positive correlation between saving and growth which is detected across time and countries. Moreover, this relationship is due to causality which runs from growth to savings, not vice versa.

The paper is organized as follows: section 2 verifies the relationship between growth and saving and explanations, given by economic theory, for this empirical fact are summarized. Section 3 introduces the model. Section 4 shows implications of the model and its differences from the basic growth model. Section 5 discusses about results and some caveats are also given. Finally, section 6 concludes this paper.

2 The theoretical and empirical relationship between saving and growth

Even if positive relationship between the growth rate of gross domestic product and national saving rate is well know we firstly present some figures to prove and emphasize the existence of this relationship. We do not give any review on the literature which has discovered and analyzed this relationship, but we use some data to show the existence of the association between growth and saving rates. Secondly, it is shortly discussed how this relationship is interpreted in economic theory.

is running from growth to savings. Probably causality is running both directions, but it may well be that the primary channel which causes the correlation is driven by causality which running from growth to savings.

2.1 The empirical relationship

Saving rate and growth are positively correlated across cross-section and time. To emphasize this relationship we use data from 18 countries[2] and time span of the data goes from 1950 to 2005.[3] We use yearly data and also ten year averages.

In figure 1 the saving and growth rates are plotted against each other with a regression line.

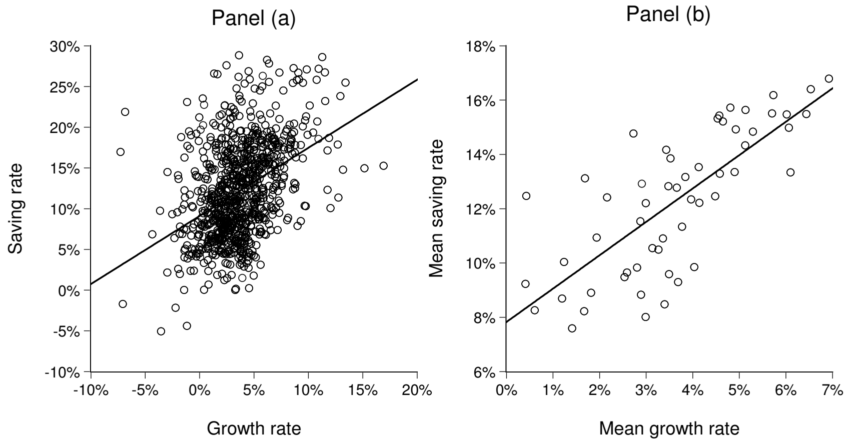

Figure 1: In the panel (a) full sample is used to verify the positive relationship between growth and saving (obs. 1062). In the panel (b) cross-section means from every year on saving and growth rates confirms the relationship given by the panel (a).

From figure 1 we can firmly conclude that saving and growth rates are positively correlated across time. Especially, the panel (b) shows a very clear positive association between growth and saving across time.

Now ten year averages are used to provide long run coefficients. By using averages we hope that the fluctuation of the business cycles are

[2] Countries are Australia, Austria, Belgium, Canada, Finland, France, The United Kingdom, Germany, Italy, Japan, Korea, The Netherlands, Norway, Portugal, Spain, Switzerland, Sweden, The U.S.A. Data source: Years 1955-1995 are from United Nations (1958), United Nations (1976), United Nations (1986) and United Nations (1999), and years 1995-2007 are from Organisation for Economic Co-operation and Development (2007). I like to thank Sami Partanen for providing data.

[3] The panel is unbalanced and in many cases data is available from 1955 to 2004.

abstracted, and hence, we capture the long run relationship. Moreover, the focus is on the cross-section dimension. Ten year averages are plotted against each other at every decade. However, there many missing observations at the first and the last decade so we restrict our focus only to 1960s, 1970s, 1980s and 1990s.

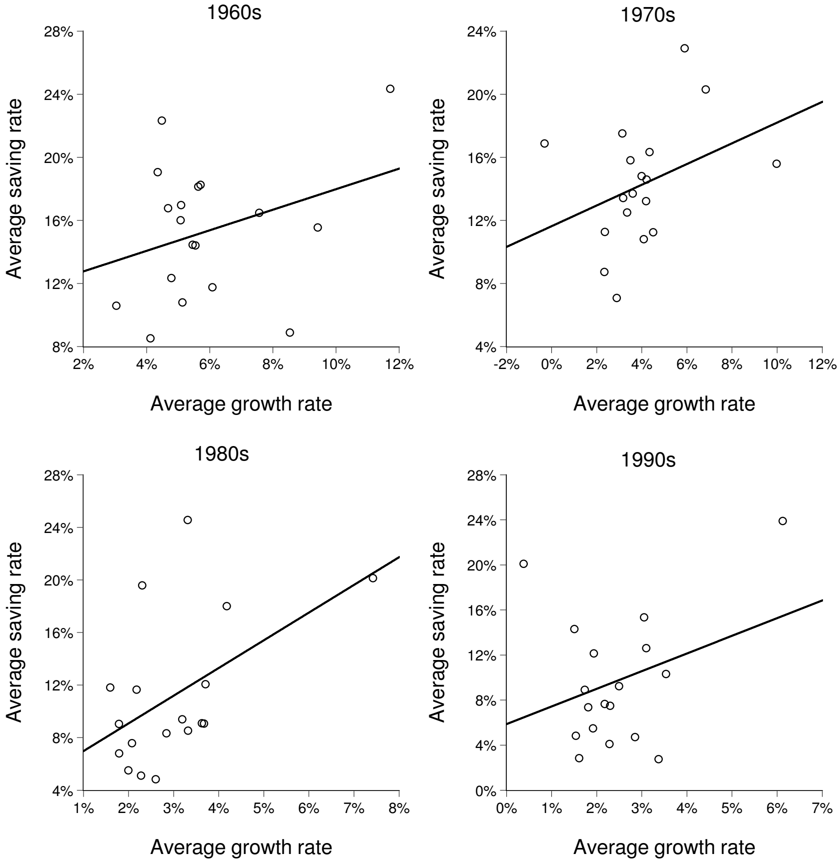

Figure 2: The cross-sectional relationship of ten-year averages of growth and saving rates from 1960s, 1970s, 1980s and 1990s.

Moreover, from figure 2 we can firmly conclude that across cross-sections growth and saving are also positively correlated and this correlation does not change in time. Hence, the empirical fact that the growth rate of GDP and the saving rate of country are positively correlated across the time and cross-sections is verified. However, even if this positive relationship is well know among economist at least from the 1960s the interpretation of that correlation is still controversial.

2.2 Interpretation in theory

Firstly, consider a case where a parameter or policy change affects households saving. One example could be that households subjective time preference declines. By using a Solow-style (Solow, 1956) growth model, an increase in saving will increase investments, and hence, capital stock moves to new steady state. Caused by the transition dynamics of this process, we observe in data a positive correlation between saving and growth. Moreover, an increase in saving can also lead to permanent increase in growth, if a Rebelo-style model (Rebelo, 1991) or endogenous growth model is used. Even if saving rate is endogenous variable, i.e. saving is a result of households optimizing behavior (Ramsey-Cass-Koopmans-style growth model), previous statement holds.

However, new and growing empirical evidence suggest that causality is not running from saving to growth but vice versa. It is shown by Carroll and Weil (1994) and conformed by Attanasio, Picci, and Scorcu (2000) that growth Granger causes saving, but saving does not Granger cause growth. Hence, the traditional interpretation of relationship between growth and saving is wrong or at least it is overstated.

Consider now that the growth rate of total factor productivity increases, hence causality is now running from growth to saving. When causality is running to this direction, as empirical evidence is suggesting, it is problematic for models based on the Ramsey-Cass-Koopmans economy. If households are optimizing their consumption (under certainty or certainty equivalence), then higher growth will lead to lower saving under reasonable parameter values.[4] The Ramsey-Cass-Koopmans growth model gives a firm foundation for association between growth and saving but it must be expanded somehow, in order to improve its ability to confront empirical facts. Obviously, in economic theory there have been many attempts to explain the dynamics of growth and savings when causality is running from growth to savings.

Firstly, in an overlapping generation setup Modigliani (1970) and Modigliani (1986) was arguing that relationship between growth and saving is indeed a result of the process where causality is running from growth to saving. That is, young households, whom are saving phase of their

[4] This result is emphasized in a partial equilibrium by Cambell (1987).

life cycle are wealthier (as a whole), due to economic growth, than old households, whom are dissaving phase of their life cycle. Hence, economy saving as a whole is positively correlated with growth. However, Modigliani's result depend on the assumption that growth only affects level of income between the generations, but shown by Carroll and Summers (1991) data implies just a opposite. Moreover, a long time ago Tobin (1967) has proved that the life cycle framework often imply negative relationship between growth and saving. We can conclude that the result of life cycle framework is a function of its special assumptions, it is not a general implication of optimal intertemporal choice.

More recent attempt to explain relationship between saving and growth in a standard growth model was given by Carroll, Overland, and Weil (2000), where authors argued that habits explain this empirical fact. Habits fit in macro data, but micro evidence is inconclusive, see Dynan (2000). Habits may well be right explanation, but it not a superior explanation, since convincing micro evidence about habits formation is still missing.

We argue that precautionary saving motive can be a key to understand this phenomena. Precautionary saving behavior created by prudence (Carroll, 1992) or by liquidity constraints (Deaton, 1991) may explain many features which are not explained in literature under certainty or certainty equivalence, see Carroll (1997). We are arguing that it is a relevant factor also when dynamics of growth and saving is considered. Precautionary saving motive has been studied significantly in general equilibrium, but models are very complicated and the relationship between growth and saving is not studied.[5] Moreover, Carroll (1997) show in a partial equilibrium model that consumption growth is equal to income growth at the steady state and Deaton (1999) argues that generally precautionary saving motives imply that saving and growth should have a slightly positive association but general equilibrium evidence is missing. This paper tries to give *a simple* model which shows that precautionary saving motive is the underlying reason why growth and saving are positively correlated across time and countries.

[5] Most well know articles are Aiyagari (1994) and Krussell and Smith (1998), see also Ljungquist and Sargent (2004) The main goal of these papers is to create a model which will explain the wealth distribution of the economy.

3 The model

The model is the basic Ramsey-Cass-Koopmans growth model but it is augmented by precautionary saving reasons.[6] The representative household does not know its life time income which causes the precautionary saving motive. The model extends the model of Toche (2005) from partial equilibrium to the general equilibrium.

The economy consist of many infinitely living households. There are two states where household can be, namely, "productive" and "unproductive".[7] First household is productive but faces in that state a probability μ of be relegated to unproductive state. The relegation process follows *a Poisson process* with arrival rate μ. This is the only source of uncertainty in this model. This relegation is a permanent one (key assumption) and precautionary saving reasons disappears at once, when a representative households' behavior equals to behavior in the standard growth model.

3.1 Demography

Assume that at time $t = 0$ the size of productive workers is nL_0 and this population is growing at rate n and there are no unproductive workers. Without loss of generality it is assumed that $L_0 = 1$. For the sake of simplification it is assumed that only productive households can have new members to their household.[8] Thus, the whole population size is

$$P(t) = \int_{-\infty}^{t} ne^{ns} ds = e^{nt} \tag{1}$$

Every moment there is a probability μ for each household be relegated to the unproductive state, hence the size of productive workers is given

[6] The basic Ramsey-Cass-Koopmans model is introduced for example in Barro and Sala-i-Martin (2004).

[7] We define "productive" and "unproductive" below.

[8] The assumption that when $t = 0$ there are no unproductive workers and the assumption that unproductive workers can not have new members their household can be easily relaxed. Note, that no new households are born. Hence, this is a representative agent model.

by

$$L(t) = \int_{-\infty}^{t} n e^{ns} e^{-\mu(t-s)} ds = \frac{n}{n+\mu} e^{nt} \\ = \theta e^{nt} \tag{2}$$

where $\theta = \frac{n}{n+\mu}$.

The size of unproductive households is given by

$$Z(t) = P(t) - L(t) = \frac{\mu}{n+\mu} e^{nt} \\ = \lambda e^{nt} \tag{3}$$

where $\lambda = \frac{\mu}{n+\mu}$.

3.2 Production

The production side of economy is just Neoclassical.[9] Hence, output of economy is given by

$$Y(t) = F\left[K(t), L(t), Z(t), T(t)\right] \\ = K(t)^{\alpha} \left[T(t)L(t)\right]^{\beta} \left[T(t)Z(t)\right]^{1-\alpha-\beta} \tag{4}$$

where $K(t)$ is capital and $T(t)$ is knowledge or technology which grows at rate g when $T(t) = T_0 e^{gt}$. Now, to define productive and unproductive the following restriction must hold: $1 - \alpha > \beta > \frac{1-\alpha}{2}$. Then $L(t)$ is "productive" labor and $Z(t)$ is "unproductive" labor. The model focuses on household behavior and not so much on production side of the economy which has been extensively examined by past growth literature. The simplest Cobb-Douglas production function is chosen but endogenous growth model could be used as well.

The focus in on a household behavior where a need for creating the precautionary saving motive, two different state are needed. These two different states are also needed in production for the sake of closing the model. Hence, we are not so interested in the effects of this two groups on production, but more or less this production function is chose to close the model an easiest possible way. We emphasize that the chose of production function is not responsible for the result, and that is why,

[9] It is assumed that production function obeys following assumptions: 1. constant returns to scale, 2. positive and diminishing returns, 3. Inada conditions.

it is kept in its most basic form, given the requirements that arose from the need to model precautionary saving behavior for households.

The representative firm's flow of profits is given by

$$\begin{aligned} \pi(t) &= F\left[K(t), L(t), Z(t), T(t)\right] - \left[r(t) - \delta\right] K(t) \\ &\quad - W_L(t)L(t) - W_Z(t)Z(t) \end{aligned} \tag{5}$$

where δ is depreciate rate of capital when $r(t)$ is real return on capital, and let $R(t) = r(t) + \delta$ be the rental price for a unit of capital services. The $W_L(t)$ and $W_Z(t)$ are wage paid for productive and unproductive workers.

Variables are written in intensive form,[10]which gives the output of economy as

$$y_t = \Lambda k_t^{\alpha} \tag{6}$$

where $\Lambda = \theta^{\beta}\lambda^{1-\alpha-\beta}$

At competitive market the inputs of production are paid by their marginal products. Thus, firms maximization problem yields in intensive form

$$r_t = \alpha\Lambda k_t^{\alpha-1} - \delta \tag{7}$$

$$\theta w_{l,t} = \beta\Lambda k_t^{\alpha} \tag{8}$$

$$\lambda w_{z,t} = (1-\alpha-\beta)\Lambda k_t^{\alpha} \tag{9}$$

3.3 Households

Households' problem can be solved recursively since the regulation from the productive state to the unproductive is a permanent one. The only source of uncertainty is the timing of regulation, when the life time income of household is uncertain.

First a representative household solves the problem of utility maximization when it is unproductive, and given that solution, it solves problem when it is in the productive state. This method was given in Toche (2005) and a permanent transition between the states is the key assumption, which enables the use of this procedure.

[10] Intensive form means here that variables are written per effective capita. That is, variables are scaled by term $[T(t)P(t)]^{-1}$.

Poisson processes are appropriate for rare stochastic events, such as job loss, and it obviously is not a good way to model general uncertainty of labor income, but here we just used it to make the life time income of household uncertain.

3.3.1 The problem of households in the unproductive state

The representative household faces following problem where it chooses flow of consumption $c_{z,t}$

$$\max \quad U \quad = \quad \int_0^\infty e^{(n-\rho+(1-\sigma)g)t} \frac{c_{z,t}^{1-\sigma}}{1-\sigma} dt \tag{10}$$

Households has CRRA utility where σ is the coefficient of relative risk aversion (or σ^{-1} is the intertemporal elasticity of consumption); ρ equals to the rate of pure time preference.[11]

Household supply labor inelastically and evolution of their assets is described by

$$\dot{a}_t = (r_t - g - n)a_t + \lambda w_{z,t} - c_{z,t} \tag{11}$$

where $\dot{a}_t = \frac{d}{dt} a_t$ which is the time derivate of assets. We also assume that the present value of assets must asymptotically be nonnegative:

$$\underset{t\to\infty}{lim} \left\{ a_t e^{-\int_0^t [r(v)-g-n]dv} \right\} \geq 0 \tag{12}$$

The first order conditions gives the usual Euler equation for consumption

$$\frac{\dot{c}_z}{c_z} = \frac{r-\rho-\sigma g}{\sigma} \tag{13}$$

and transversality condition is

$$\underset{t\to\infty}{lim} \left[a_t e^{-\phi_t} \right] = 0 \tag{14}$$

where $e^{-\phi_t} = \exp\left\{ - \left[\int_0^\infty \frac{1}{t} r(v) dv - g - n \right] t \right\}$.

[11] For the sake of utility to be bounded when $c_{z,t}$ is constant it is assumed that $n - \rho + (1-\sigma)g$ must be negative.

Moreover, consumption follows the standard permanent income result

$$c_{z,0} = m_t^{-1}\left[a_t + w_{z,0}\right] \tag{15}$$

where

$$w_{z,0} = \int_0^\infty \lambda w_{z,t} e^{-\phi_t} dt \tag{16}$$

$$m_t = \int_0^\infty \exp\left\{\left[\int_0^t \frac{1}{t}(1-\frac{1}{\sigma})r(v)dv + n - \frac{\rho}{\sigma}\right] t\right\} dt \tag{17}$$

3.3.2 The problem of households in the productive state

A representative household has now solved its problem when it is in unproductive state. Given that solution i.e. equation (15) it solves problem in the productive state. So, household chooses an expected flow of its consumption $c_{l,t}$ and $c_{z,t}$, where $c_{z,t}$ is given and equals to $c_{z,0}$. A representative household problem is given by

$$\max \quad E_t U = E_t \int_0^\infty e^{(n-\rho+(1-\sigma)g)t}\left[\frac{c_{l,t}^{1-\sigma}}{1-\sigma} + \frac{c_{z,0}^{1-\sigma}}{1-\sigma}\right] dt \tag{18}$$

where E_t is conditional expectation operator. Problem can be rewritten

$$\max \quad U = \int_0^\infty e^{(n-\rho+(1-\sigma)g-\mu)t}\left[\frac{c_{l,t}^{1-\sigma}}{1-\sigma} + \mu\frac{c_{z,0}^{1-\sigma}}{1-\sigma}\right] dt \tag{19}$$

The flow of household assets can be given in the same manner as in unproductive state[12]

$$\dot{a_t} = (r_t - g - n)a_t - w_{l,t} - c_{l,t} \tag{20}$$

but the only difference is now that $w_{l,t}$ is uncertain.[13] In addition, assets at the productive state must full fill equation (15), hence, at every t following condition must hold

$$c_{z,0} = a_t + w_{z,0} \tag{21}$$

[12] For the sake of utility be bounded it is assumed that $n-\rho+(1-\sigma)g-\mu$ must be negative.

[13] Assets must asymptotically be nonnegative, hence similar condition as equation (12) can be written now as $\lim_{t\to\infty}\left\{a_t e^{-\int_0^t [r(v)-g-n+\mu]dv}\right\} \geq 0$

By using equations (19), (20) and (21) the maximization problem of the representative household in the productive state can be constructed. Lagrangea-Hamiltonian equation can be written as

$$\begin{aligned} L(c_{l,t}, c_{z,0}, a_t, \nu_t, \upsilon_t) &= e^{(n-\rho+(1-\sigma)g-\mu)t}\left[\frac{c_{l,t}^{1-\sigma}}{1-\sigma} + \mu\frac{c_{z,0}^{1-\sigma}}{1-\sigma}\right] \\ &\quad +\nu_t\left[(r_t - g - n)a_t - w_{l,t} - c_{l,t}\right] \\ &\quad +\upsilon_t(a_t + w_{z,0} - c_{z,0}) \end{aligned} \tag{22}$$

First order conditions gives the Euler equation for productive households'

$$\frac{\dot{c}_l}{c_l} = \frac{r - \rho - \sigma g + \mu\left[\left(\frac{c_l}{c_{z,0}}\right)^{\sigma} - 1\right]}{\sigma} \tag{23}$$

The term $\left[\left(\frac{c_{l,t}}{c_{z,0}}\right)^{\sigma} - 1\right]$ describes household precautionary saving motive, and given that $c_l > c_{z,0}$, the term is unambiguously positive.[14] It implies that the higher is μ the higher is consumption growth and the cause of higher consumption growth is given by precautionary motive. The constant risk of regulation to unproductive state, which causes a lower labor income, gives household a precautionary saving motive.

Another matter which affects to precautionary saving motive is the ratio of consumptions' levels between the states. Moreover, precautionary saving arose from differences in marginal utilities, if marginal utilities, or consumption levels, equals between the states precautionary saving motive disappears. Larger is the gap between consumptions' levels, i.e c_l and $c_{z,0}$, higher will be precautionary saving, since consumer wants to smooth consumption between differ states. The level of consumption in different states is a function of wage paid in those states – controlled by parameter β – and capital income. On the one hand, households want to increase their consumption in the productive state, since they want utilize high wage their are receiving. On the another hand risk of being regulated to the unproductive state reduces their desire to consume all their income. Extra savings done in the productive state are needed for consumption in the unproductive state.

[14] Transversality condition is now $\underset{t\to\infty}{lim}\left\{a_t \exp\left[-\phi_t - \mu\left(\frac{c_{l,t}}{c_{z,0}}\right)^{\sigma} t - \mu t\right]\right\} = 0$. Comparing limiting conditions for assets and transversality condition between two states we can conclude that if these conditions hold in the unproductive state, as they must in steady state, they will hold also in the unproductive state. Therefore, only the unproductive state limiting and transversality conditions are necessary.

Note that, if $\mu = 0$ the Euler equations and the whole maximization problem comes back to basic maximization problem of representative household in the ordinary growth model. That is, the Euler equation (23) in productive state equals to Euler equation in the unproductive state, that is equation (13).

3.4 Aggregation

Next task is to aggregate the behavior of households in the productive state and in the unproductive state. Aggregation is simple since aggregate variables are just given by the number of agent in each state times variable in interest.

We start with assets market where supply and demand of assets must cancel within the economy. Evolution of aggregate assets is

$$\begin{aligned} \dot{A}(t) &\equiv L(t)\dot{A}(t) + Z(t)\dot{A}(t) \qquad (24) \\ &= r(t)L(t)A(t) + r(t)Z(t)A(t) + L(t)W_L(t) + Z(t)W_L(t) \\ &\quad -L(t)C_L(t) - Z(t)C_Z(t) \end{aligned}$$

Equation (24) can be written in intensive form which gives

$$\dot{a_t} = (r_t - g - n)a_t + \theta w_{l,t} + \lambda w_{z,t} - \theta c_{l,t} - \lambda c_{z,t} \qquad (25)$$

Equilibrium in the assets market requires $a_t = k_t$ and equations (6), (7), (8) and (9) can be used to generate the equation of motion of capital

$$\dot{k_t} = \Lambda k_t^{\alpha} - \theta c_{l,t} - \lambda c_{z,t} - (\delta + n + g)k_t \qquad (26)$$

An aggregate consumption is formed in the same manner, therefore, the intensive form of aggregate consumption is

$$c_t \equiv \theta c_{l,t} + \lambda c_{z,t} \qquad (27)$$

The evolution of the aggregate consumption is given by

$$\dot{c_t} = \theta \dot{c}_{l,t} + \lambda \dot{c}_{z,t} \qquad (28)$$

Use the Euler equations (13) and (23) from unproductive and productive states to rewrite the evolution of aggregate consumption

$$\begin{aligned} \dot{c_t} &= \frac{r_t - \rho - \sigma g + \mu \left[\left(\frac{c_{l,t}}{c_{z,0}} \right)^{\sigma} - 1 \right]}{\sigma} \theta c_{l,t} \qquad (29) \\ &\quad + \frac{r_t - \rho - \sigma g}{\sigma} \lambda c_{z,t} \end{aligned}$$

and use condition in equation (27) and equation (28) to restate the evolution of aggregate consumption and capital

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho - \sigma g}{\sigma} + \frac{\mu}{\sigma}\left[\left(\frac{c_{l,t}}{c_{z,0}}\right)^{\sigma} - 1\right]\frac{\theta c_{l,t}}{c_t} \tag{30}$$

$$\dot{k}_t = \Lambda k_t^{\alpha} - c_t - (\delta + n + g)k_t \tag{31}$$

Equations (30) and (31) with equation (15) determines the time paths of c and k.[15]

3.5 The steady state

At the steady state c_t and k_t must be constants. Thus, we solve $c_\star$ from (31) as a function of parameters and $k_\star$. Equation (30) determines $k_\star$ as a function of parameters, but $c_{l,t}$ is unknow. We can use the definition (27) to solve $c_{l,t} = \theta^{-1}(c_t - \lambda c_{z,t})$ and substitute this to equation (30), which gives

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho - \sigma g}{\sigma} + \frac{\theta\mu}{\sigma}\left[\left(\frac{c_t}{c_{z,0}} - \frac{\lambda}{\theta}\right)^{\sigma} - 1\right]\left[\frac{1}{\theta} - \frac{\lambda c_{z,0}}{\theta c_t}\right] \tag{32}$$

The steady state imply that left hand side of equation (32) is zero, and we can solve $k_\star$, since at the steady state we can write this equation as follows

$$\frac{\alpha\Lambda k_\star^{\alpha-1} - \delta - \rho - \sigma g}{\sigma} + \frac{\theta\mu}{\sigma}\left[\left(\frac{c_\star}{c_{z,\star}} - \frac{\lambda}{\theta}\right)^{\sigma} - 1\right]\left[\frac{1}{\theta} - \frac{\lambda c_{z,\star}}{\theta c_\star}\right] = 0 \tag{33}$$

where equation (31) defines $c_\star$ by

$$c_\star = \Lambda k_\star^{\alpha} - (\delta - g - n)k_\star \tag{34}$$

and $c_{z,\star}$ is given by equation (15)

$$c_{z,\star} = \left[\rho - \sigma n + \frac{1}{\sigma}(1-\sigma)\left(\alpha\Lambda k_\star^{\alpha-1} - \delta\right)\right] \times \left[k_\star + \frac{\lambda(1-\alpha-\beta)\Lambda k_\star^{\alpha}}{\alpha\Lambda k_\star^{\alpha-1} - (\delta + g + n)}\right] \tag{35}$$

[15] Obviously, initial condition k_0 and tranversality condition, equation (14), is needed also to determine the time paths of c and k.

By substituting conditions (34) and (35) to (33) we can solve $k_\star$ as a function of parameters. Expression is quite complicated so we have to use some numerical method to solve it. However, if we restrict our parameters in way that $\sigma = 1$ and $w_{0,\star} = 0$ it is possible to solve $k_\star$ without numerical methods, and model is tractable.

Equations (32) and (31) are linearized by taking first order Taylor approximation a round the steady state. System of linearized equations are used for the analysis of nature of steady state and to find out the time paths of c_t and k_t. There are often several steady states, but only steady states, which are a) stable, b) full fill transversality condition and c) gives a positive values for consumption and saving, are considered. Then the situations of possible multiple equilibria are avoided. Finally, we are interested in saving rate which is defined as

$$s_t = 1 - \frac{c_t}{y_t} \tag{36}$$

4 Implications of models for saving and growth

Now we are ready to show implications of the model introduced in section 3 and its differences to the basic Ramsey-Cass-Koopmans model. However, before going to results some discussion about choices of values of parameters is needed. The purpose of simulations is compare the model introduced in this paper to the basic Ramsey-Cass-Koopmans model. In order to do that, we choose our parameters by following basic graduate book on economic growth (for example, Barro and Sala-i-Martin (2004)).

The experiment which we are considering here is an unexpected increase in the growth rate of productivity . There are many other scenarios which also affect to growth and saving rates. This scenario is not used generally in traditional growth literature, but this scenario can be interpreted in a case where we consider broadly defined technology (including, for example, property rights and restrictions in trade). We consider here a permanent change, but we do not argue that growth rates of technology would permanently differ each other between countries. However, when changes in g takes a sufficiently long time, the results would be similar than in the case of temporary increase.

4.1 The implications of Ramsey-Cass-Koopmans growth model for saving and growth

First we choose parameters values for the basic Ramsey-Cass-Koopmans model: let $\delta = \rho = 0.07$; $n = 0.01$; $g = 0.01$ and σ is between range 1-3. The value of ρ must be quite high in order to avoid a situation where assets would grow to infinity for households who has precautionary saving motive. This "impatience" assumption is highlighted for example by Deaton (1991).[16] Moreover, as we mentioned in section 2 that the effects of growth on saving rate depends on parameters values, it strongly depends on values of σ^{-1}, i.e. the elasticity of intertemporal substitution. Another parameter which is important is α. The traditional choice of $\alpha = 1/3$, but as argued by Mankiw, Romer, and Weil (1992) it seems to be too low.[17] Following their proposal we use the value $\alpha = 2/3$, hence, capital also includes human capital.[18]

Consider now the parameters values given above and the value of $\sigma = 1$. That is, we assume log-utility, which is often used in the literature to describe consumers behavior. Figure 3 shows the time paths of growth rate of GDP and saving rate.

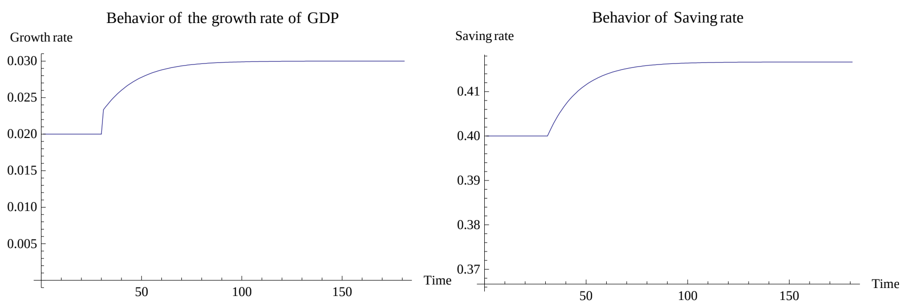

Figure 3: At $t = 30$ g increases from the value 0.01 to the value 0.02 when $\sigma = 1$.

Clearly, we can now conclude that even when causality is running from growth to savings we have a positive relationship between growth and saving. However, widely accept parameter values of σ are much higher

[16] Unfortunately, we can not give any "impatience" condition in this model.

[17] Obviously, we must assume now production function is in the basic Cobb-Douglas form: $Y(t) = K(t)^{\alpha}L(t)^{1-\alpha}$ and the Euler equation is given by equation (13).

[18] Once again, we like to compare our results to existing ones and these parameters values are usually used (for example in Barro and Sala-i-Martin (2004)).

than 1.[19] Consider now a value $\sigma = 3$, which is more reasonable than 1. The same experiment is done as previously and figure 4 gives the time paths.

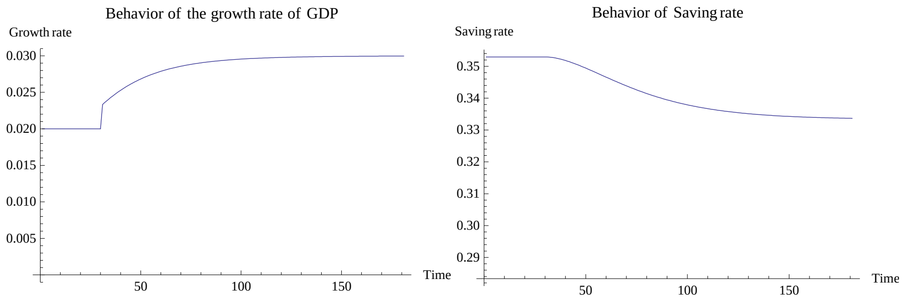

Figure 4: At $t = 30$ g increases from the value 0.01 to the value 0.02 when $\sigma = 3$.

Now the correlation between saving rate and growth is a negative one. Hence, a positive relationship between growth rate and saving rate is only possible for values of the elasticity of substitution which are not plausible given empirical evidence. That is why we need to augment the basic Ramsey-Cass-Koopmans model somehow, or in another words: a range of circumstances which provide a positive association between growth and saving rate must be much greater than the circumstances of the basic Ramsey-Cass-Koopmans growth model.

4.2 Implications of the model for saving and growth

The basic Ramsey-Cass-Koopmans growth model does not imply with plausible parameter values a positive relationship between growth and saving rate. However, we argue that if the basic growth model is augmented by precautionary saving motive a positive relationship can be generated with plausible parameter values.

Before going to the results we must define some parameter values. The expected time in the productive state is given μ^{-1}. Unfortunately we can not give any good estimate for μ. However, we show that even small values of μ can have significant effects on behavior of saving rate. Moreover, high values of μ can create an unreasonable large precautionary motive, since then households are only a short time in

[19] Hall (1988) found a minimum estimate for σ which was 5.

the productive state but infinitely in the unproductive state. Hence, it is optimal to save almost all income in productive state. Savings then yield a capital income which raises consumption and utility in the unproductive state where household is infinitely. As a result, we set almost arbitrarily $\mu = 0.001$. However, the correlation between growth and saving do not depend on the value of μ, only the level of consumption and saving in a different states are.

Secondly, we have to choose the value of β. For sake of keeping precautionary saving motive reasonable we keep level of income between the states small, hence given that $\alpha = 2/3$, we assume that $\beta = 0.17$.[20] Other parameters are kept in the values which were given above.

Now we redo our experiment where the growth of productive increases and the value of $\sigma = 3$. Figure 5 shows the result.

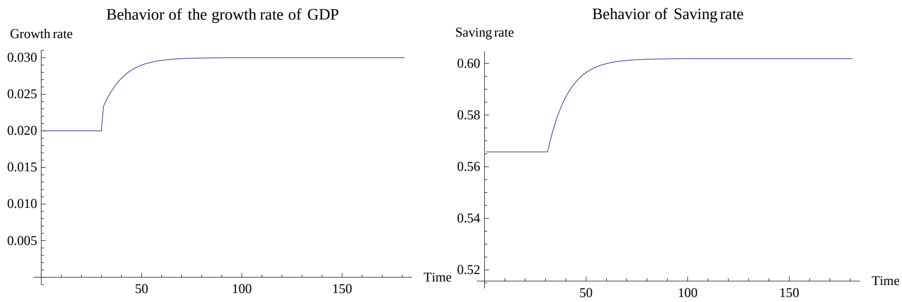

Figure 5: At $t = 30$ g increases from the value 0.01 to the value 0.02 when $\sigma = 3$ and $\mu = 0.001$.

When the basic Ramsey-Cass-Koopmans growth model is augmented by precautionary saving motives we can explain with plausible parameter values the positive correlation between saving and growth rates. This result is robust for changes in any parameter values. So, these simulations suggest that precautionary saving motives can be important when responses of saving rate respect to growth rate are considered. Moreover, our augmented model significantly expands the range of circumstances where the basic growth model predicts a positive relationship between saving and growth rates.

As we mentioned above the value of μ does not change the results. Let $\mu = 0.025$ which could be considered as a reasonable value, since

[20] The value of β is not significant in any way when results are considered, all what matters is the value of μ.

the expected time in the productive state is then 40 years and this parameter value were used by Toche (2005). Figure 6 shoes the time paths.

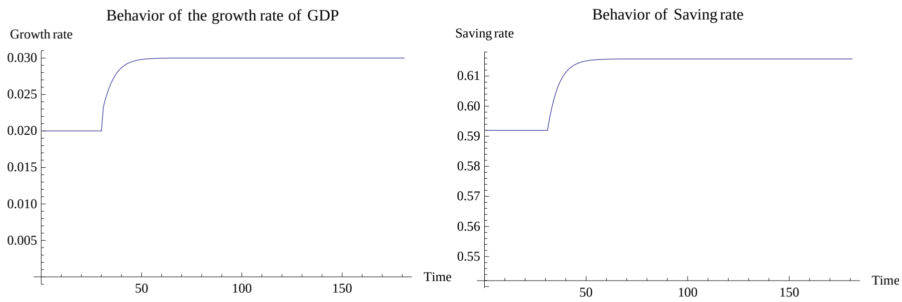

Figure 6: At $t = 30$ g increases from the value 0.01 to the value 0.02 when $\sigma = 3$ and $\mu = 0.025$.

Only the speed of convergence is higher than previous cases, otherwise the results are more or less the same.

5 Discussion and caveats

For describing the differences between the two models a collection of some key variables from the basic growth model (i.e. Ramsey-Cass-Koopmans model) and the model presented in this paper (the augmented model) are presented in table 1. Values in the table 1 are from the cases which time paths of saving and growth rates were given in figures 4 and 5.

Obviously, the exact numbers should not been taken too seriously, since models are very simple. However, we can combine the numbers to each other. Saving rates between the two models are quite different, hence the inclusion of precautionary saving motive into the basic growth model cause high saving rate, even if uncertainty in the model is quite low. Moreover, the real interest rate differs also significantly, since capital stock – due to precautionary saving – is larger than the stock in the basic model. This feature is consistent with evidence from high saving countries as Japan, where stock market returns have been quite low at 1970s when saving rate were high. Finally, including uncertainty to the basic growth model increases greatly the speed of convergence. This feature is consistent with previous simulation results (see, Carroll

Table 1: Some key values at the steady states from the models when $\sigma = 3$ and $\mu = 0.001$

Variables	The basic growth model		The augmented model	
	Value at $t = 0$	Value at $t_\star$	Value at $t = 0$	Value at $t_\star$
s	35%	33%	57%	60%
r	10%	13%	3.6%	4.1%
k	60.3	37.0	73.1	64.1
$-\psi$		0.033		0.103

Explanations for the table: s is the saving rate; r is the real interest rate; k is the capital in its intensive form and $-\psi$ gives the speed of convergence. The column of t_0 gives the values of variables when parameters are their baseline values. The column of $t_\star$ gives the values of variables when $g = 0.02$.

(1997)). However, the ultimate question is: which effect cause a positive correlation between saving and growth in the augmented model?

The reason is simple. Higher growth in future increases the ratio between $\frac{c_{l,t}}{c_{z,t}}$. Thus, households in the productive state must increase their savings or buffer-stock to smooth their consumption between the two states. Moreover, higher capital stock also imply lower interest rate, when the interest rate – or its shift derived by exogenous shock – is smaller, then the income effect is also smaller. Hence, in the augmented model the substitution effect is stronger than in the basic model which partly explains the result. The model shows that precautionary saving motive can be the underlying reason for the positive association between growth and saving. However, some caveats must be given, since the model includes some features which are not so harmless. We have two things that disturb us.

Firstly, every household in the model holds the same amount of capital, after all this is a representative agent model, and this is a necessary condition for a simple aggregation rule. But it would be more realistic to think that households who has a precautionary saving motive would hold a differ amount of capital than households which are behaving according to permanent income model. So, to confirm a role of precautionary saving in growth saving relationship more complicated model is needed.[21] We could use a model given in Blanchard (1985), but then marginal propensity to consume would be depend on age. Since, under uncertainty marginal propensity to consume would be a function of assets which are different in each age. When marginal propensity to consume depend on age the aggregation is very difficult.

Secondly, we could not find any parameter restrictions which would ensure the existence of stable equilibrium. Easily we are in a situation that there is no equilibrium for the system which full fill transversality condition. So, we can not give any parameter ranges where the model has a unique stable equilibrium, but every case have to be considered on its own.

6 Conclusions

Precautionary saving can be an answer for many puzzling features detected from the time series data of consumption and income. The positive association between growth rate of GDP and the saving rate is a well known empirical fact, and growing evidence suggest that causality is running from growth to savings not vice versa as traditional hypothesis argues. The standard Ramsey-Cass-Koopmans growth model provides a firm theoretical foundation why saving should affect growth, but under plausible parameter values higher growth in the future should lead lower saving. Hence the positive correlation between growth and saving cannot be explained by the Ramsey-Cass-Koopmans growth model. The model presented in the paper has two contributions.

Firstly, the model introduced in this paper showed that precautionary saving motive maybe the underlying reason for the positive relationship between growth and saving when the Ramsey-Cass-Koopmans type model is augmented by precautionary saving motives. However, more complicated or realistic model is needed to verify conclusion made in the paper, but given shortcomings of the model, we can conclude that precautionary saving motive can explain the correlation between growth and saving rates. That is, precautionary saving motive increases saving rate and capital stock in economy which imply a lower interest rate and smaller shift in interest derived by exogenous shock. Hence,

[21] This kind of model would imply state space system which would be hard to solve. However, a model in line with Aiyagari (1994) and Krussell and Smith (1998) is needed.

income effect becomes smaller than in the basic model. This effect and precautionary motive itself cause the positive relationship between growth and saving. Moreover, we gave a reasonable explanation why saving and growth rates are positively correlated in the data, even if the causality is running from growth to saving. So, the traditional channel (from saving to growth) – explained by traditional growth model – may not be even the primary reason for this positive relationship.

Secondly, the paper introduced a simple manner to solve a general equilibrium model when there are two type of agents and there is a stochastic transfer process between the groups. This kind of model could be useful in the models of public economics, where the models of two types of agents are often used.

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