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ABSTRACT

We study farsighted stability under the assumption that coalitions may condition actions on the history of the play. vNM stable set over possible play paths is defined with respect to the indirect dominance relation. We show that such dynamic stable set always exists. It is characterized by a generalization of the ultimate uncovered set. In unbounded agenda setting context, the dynamic stable set implements only efficient outcomes if active coalitions form a majority.

JEL Classification: C71, C72

Keywords: vNM stable set, farsightedness, dynamic, agenda formation

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1 Introduction

In the literature of coalition formation it is standard to focus on *stationary* coalitional strategies, i.e. strategies that depend only on the status quo, and not on the history of how the status quo has been reached. This strategic rigidity contributes to the various difficulties in determining a solution. Indeed, as demonstrated by Barberà and Gerber (2007), there is no Pareto-optimal, stable, farsighted, and self-consistent coalition formation solution even in a restricted class of games when strategies are stationary.[1] This paper studies the scope of solutions that become feasible when more general strategies allowed.

We develop model of stable and farsighted coalitional decision making in the general framework by Chwe (1994). Our solution is based on the von Neumann-Morgenstern (vNM) stable sets. To capture players' farsightedness, we follow Chwe (1994) and Harsanyi (1970), by using *indirect dominance* as the dominance criterion.[2] The novel feature of the solution is that it explicitly takes into account dynamics by introducing histories, i.e. sequences of status quos that have been visited before reaching the current status quo, into the definition of feasible outcomes. A *dynamic stable set* is simply a vNM stable set defined over the set of histories. Hence, stability of a status quo is conditioned on the history via which it is reached.

It is well known that a stable set may fail to exists with stationary strategies, even when defined with respect to indirect dominance. In fact, Chwe (1984) and Barberà and Gerber (2007) suggest that no solution concept can satisfy internal stability in domination-rich environments. We show that this premise is based on a narrow interpretation of the coalition formation process.

Our main result is that there always exists a dynamic stable set that is defined with respect to the indirect dominance relation. A complete characterization of the outcomes implementable via such dynamic stable sets are characterized. We show that the unique maximal dynamic stable set can be interpreted as the *ultimate uncovered set* that is defined with respect to indirect dominance.[3]

However, it can be argued that indirect dominance does not adequately capture agents' farsightedness. As Xue (1998) discusses, indirect dominance may reflect unreasonably optimistic expectations concerning the consequences of the deviation. Nothing in the notion of indirect dominance prevents the coalitions from deviating from the dominance path before reaching

[1] Barberà and Gerber (2007) focus on hedonic games.

[2] Outcome b indirectly dominates a if there is a sequence of outcomes starting from a and ending to b that are paired with potentially active coalitions such that any coalition prefers b over the outcome it is paired with.

[3] The uncovered set is due to Fishburn (1977), and Miller (1980). Iteration of the uncovered set is studied e.g. by Dutta (1988), and Coughlan and LeBreton (1999).

the final outcome.

This questions also the validity of indirect dominance as the appropriate primitive of the model. However, not in all cases is the criticism warranted. This is true, in particular, when indirect dominance *implies* direct dominance (the converse is true automatically). If the final element in any indirect dominance chain could be reached in one step, then no concerns would arise as to whether intermediate deviation from the path could take place. Thus, if indirect and direct dominance coincide, then it justified to argue that the dominance relation truly reflects farsightedness.

A particular scenario where the equivalence of direct and indirect dominance automatically holds is *endogenous agenda formation*. The rules of the agenda setting game are simple: An outcome herited from the history is on the table. Then any coalition of at least k agents (e.g. a majority) may challenge the outcome on the table by proposing (any) another outcome. The new outcome then becomes the alternative on the table. When the alternative on the table is not challenged, it is implemented.

We show that when k is at least one half of the population (i.e. a majority), then the resulting set of implementable outcomes is a subset of Pareto-optimal outcomes. The converse, however, is not true. With small k, a non-efficient outcome may become implemented.

Agenda formation has been an important topic in the literature. The distinctive feature of our approach is the unboundedness of the agenda formation process. Most of the literature assume a fixed, finitely long agenda. E.g. Moulin (1979), Shepsle and Weingast, (1984), and Banks (1985) analyse majority voting in finite elimination trees. Dutta, Jackson and Le Breton (2001a, see also 2001b, 2002) study endogenous agenda formation with bounded maximum length of the resulting agenda. With unbounded agenda formation process, backwards induction cannot be used to solve the model.[4] It needs to be identified via internal consistency considerations.

2 Dynamic stability

The game is defined by a list $\Gamma = \langle N, A, (\succeq_i)_{i\in N}, (\rightarrow_S)_{S\subseteq N}\rangle$, where N is the set of players, A is the nonempty and finite set of outcomes, $\succeq_i$ is the preference relation of player $i \in N$ with the the asymmetric part $\succ_i$. Denote $a \succ_S b$ if $a \succ_i b$ for all $i \in S \subseteq N$. A binary relation $\rightarrow_S$ on A represents what a coalition S can do: $a \rightarrow_S b$ means that S is allowed to replace the status quo outcome a with a new status quo b. Collection $(\rightarrow_S)_{S\in 2^N}$ is part of the basic data of the game and not subject to actions of the players.

The game is interpreted as follows: There is an initial status quo outcome a_0. At any stage $t = 0, 1, ...$, alternative a_t is the status quo. If the members of coalition S decide to change the status quo a_t to b, and $a_t \rightarrow_S b$, then

[4] For recent contributions towards this direction, see Penn (2005a,b).

b becomes the new status quo in stage $t+1$. If no coalition S such that $a_t \to_S b$ wants to induce b as the new status quo, then a_t is implemented and payoffs materialize.

Players only care about the implemented outcome. The status quo outcome is always public information.

The model allows many interpretations. For applications that go under the umbrella of this framework, see Chwe (1994) or Xue (1998).

We say that b dominates a, and write $b > a$, if there is a coalition S such that $a \to_S b$ and $b \succ_S a$. However, farsighted players should anticipate further deviations along the deviation path, and care only of the final outcome (see Harsaynyi, 1974). To capture farsightedness, we extend the notion of dominance as follows.

Definition 1 (Indirect Dominance) *Outcome* b indirectly dominates a, *written as* $b \gg a$, *if there are outcomes* $a = b_0, ..., b_m = b$ *and coalitions* $S_0, ..., S_{m-1}$ *such that* $b_k \to_{S_k} b_{k+1}$ *and* $b \succ_{S_k} b_k$, *for all* $k = 0, ..., m-1$.

A successive coalitional actions induce a history $a_0, a_1, ...$ of non-implemented status quos. Denote by H the set of all possible finite histories $(a_0, ..., a_k)$ such that $a_{l-1} \to_S a_l$, for some $S \in 2^N$, for all $l = 1, ..., k$.

Definition 2 (Dynamic stability) *A* dynamic stable set $V \subset H$ *is defined by:*

1. *(External dynamic stability) If* $(h, a) \notin V$, *then there is* $(h, a, ..., b) \in V$ *such that* $b \gg a$.
2. *(Internal dynamic stability) If* $(h, a) \in V$, *then there is no* $(h, a, ..., b) \in V$ *such that* $b \gg a$.

Dynamic stable set is free of inner contradictions and accounts for every history is excludes. That is, no element in the set is indirectly dominated by an element in the set and every element outside the set is indirectly dominated by an element in the set.

The final outcome a in a history (h, a) in a dynamic stable set V is interpreted to become implemented once the history is reached as no coalition wants to challenge it given that the play eventually converges to back to V. Set Y of outcomes is said to be *implementable* with the dynamic stable set V if

$$Y = \{a : (h, a) \in V, h \in H\}.$$

The set of implementable outcomes is our main object of our study. The initial status quo may affect the eventual outcome that will become implemented in the maximal set of implementable outcomes but not the set itself.

The concept of stable set aims at capturing in reduced form some aspects of sequential rationality in dynamic coalition formation. Consider any history (h, a) in the set of implementable outcomes of a dynamic stable set (referred henceforth as a "stable outcome"). Suppose that a coalition replaces a with b. Since (h, a) is stable, by external stability, the history (h, a, b) is indirectly dominated by a stable $(h, a, b, ..., c)$. However, by internal stability, $(h, a, b, ..., c)$ cannot indirectly dominate (h, a) which can only mean that c is not preferred to a by all players in the original, deviating coalition. Hence, assuming that a deviation will be followed by further dynamics along some indirect dominance path until an element in the stable set is reached, no coalition of agents can agree on challenging a stable status quo.

As is customary in much of the coalition literature, the details of the coalition formation process are out left from the model. In particular, we do not assume anything on how a coalition is grouped, why some coalition and not another is formed, and how a coalition chooses to which alternative it will move the game.[5] The outlook of the stable set may be sensitive to details. However, the existence, which we shall establish, is not.

2.1 Characterization

Denote the set of outcomes that indirectly dominate a by

$$D(a) = \{b \in A : b \gg a\}.$$

Definition 3 (Consistent choice set) *A nonempty set $X \subseteq A$ is a* consistent choice set *if, for any $a \in X$, $b \in D(a)$ implies that there is $c \in X$ such that $c \in D(b) \setminus D(a)$.*

That is, if an element in a consistent choice set X is indirectly dominated by an (any) alternative, then this alternative is itself indirectly dominated by another alternative in X that does *not* indirectly dominate the original element in X. A consistent choice set meeting Definition 3 is related to the consistent set defined by Chwe (1994). The relation between the two notions is discussed in Section 4.

Now we establish that any stable set (would such exist) is outcome equivalent to a consistent choice set.

Lemma 4 *V is a dynamic stable set only if $\{a : (h, a) \in V,\ h \in H\}$ is a consistent choice set.*

[5] However, equibrium coalition formation process of a non-cooperative model can usually be captured in a reduced form via a stable set that is defined with respect to appropriately defined dominance relation, see e.g. Diamantoudi and Xue (2006).

Proof. Let V be a stable set. We show that $\{a' : (h', a') \in V, h' \in H\}$ meets Definition 3. Take $a \in \{a' : (h', a') \in V, h' \in H\}$. Identify $h \in H$ such that $(h, a) \in V$. Suppose that there is $(h, a, ..., b)$ such that $b \gg a$. By internal stability, $(h, a, ..., b) \notin V$. By external stability, there is $(h, a, ..., b, ..., c) \in V$ such that $c \gg b$. Thus $c \in \{a' : (h', a') \in V, h' \in H\}$. By internal stability, however, not $c \gg a$, as desired. ■

Now we show the converse, that for any consistent choice set there exists a stable set that is outcome equivalent to the consistent choice set. For this purpose, we construct a stable set.

Denote the complement of $D(a)$ by

$$L(a) = \{b \in A : \text{not } b \gg a\}. \tag{1}$$

Fix a consistent choice set X. Let

$$Q = \{q^a : a \in X\}, \tag{2}$$

be a partition of H, constructed recursively as follows: Let $a_0 \in q^{a_0}$. For any $(a_0, ..., a_k) \in H$, if $(a_0, ..., a_k) \in q^a$, then

$$(a_0, ..., a_k, b) \in \begin{cases} q^b, & \text{if } b \in L(a) \cap X, \\ q^a, & \text{if } b \notin L(a) \cap X. \end{cases} \tag{3}$$

Proceeding this way for all $b \in A$, and for all $k = 0, 1, ...$, each history $(a_0, ..., a_k, b)$ in H is allocated into one element of Q.

Note that the transition rule (3) can be described directly via a transition function τ such that

$$\tau(q^a, b) = \begin{cases} q^b, & \text{if } b \in L(a) \cap X, \\ q^a, & \text{if } b \notin L(a) \cap X. \end{cases} \tag{4}$$

That is, whenever the play is in state q and the status quo is b, the new state will be $\tau(q, b)$. We will use this specification in the proof below.

We let the agents to implement a the status quo b in state q^a if b is contained in $L(a) \cap X$. The procedure V^X corresponding to this idea is defined by

$$V^X = \{(q^a, b) : b \in L(a) \cap X,\ a \in X\}.$$

By construction,

$$\begin{aligned} \left\{b : (q^a, b) \in V^X\right\} &= \{b : b \in L(a) \cap X, \text{ for some } a \in X\} \\ &= \{b : b \in \{b\} \cap X\} \\ &= X. \end{aligned} \tag{5}$$

Thus, elements in the consistent choice set X are implementable with the procedure V^X. We next show that V^X is a stable set.[6]

[6] With the notational simplification that $(h, b) \in V^X$ if and only if $(q_a, b) \in V^X$ and $h \in q^a$.

Lemma 5 *V^X is a dynamic stable set.*

Proof. *External stability*: Take any (q^a, b) such that $b \notin X \cap L(a)$. If $b \notin L(a)$, then $X \cap L(a) \setminus L(b)$ is nonempty by the definition of X. If $b \in L(a) \setminus X$, then $a \in X \cap L(a) \setminus L(b)$. Thus there is c such that $c \in X \cap L(a) \setminus L(b)$. By the construction of V^X, $(q^a, c) \in V^X$.

Internal stability: Take any (q^a, b) such that $b \in X \cap L(a)$. Then $(q^a, b) \in V^X$. Take any $(q^d, c) \in V^X$ (i.e. $c \in L(d) \cap X$). Suppose that c indirectly dominates b via sequences $b_0, ..., b_m \in A$ and $q_0, ..., q_m \in Q$ such that $\tau(a_k, q_k) = q_{k+1}$ and $b_m \succ_{S_k} b_k$ for all $k = 0, ..., m-1$, and such that $c = b_m$ and $b = b_0$.

By the construction of τ, $q_0 \neq q_1$. Let $k \leq m-1$ be the highest integer such that $q_k \neq q^d$. Since there is no transition away from state q^d after k, it must be, by the construction of τ, that $b_k = d$. Thus $c \in L(d) \cap X$ implies $b_m \in L(b_k) \cap X$. Since b_m does not indirectly dominate b_k it follows that b_m does not indirectly dominate b_0 via sequences $b_0, ..., b_k, ..., b_m \in A$ and $q_0, ..., q_m \in Q$, a contradiction to the supposition. ■

By Lemma 4, a set Y of alternatives is implementable via a dynamic stable set only if Y is a consistent choice set. Conversely, by (5) and Lemma 5, outcomes of any consistent choice can be implemented via a stable set. We compound these observations in the following characterization.

Theorem 6 *Set Y of alternatives is implementable in a dynamic stable set if and only if Y is a consistent choice set.*

this result does not, however, tell nothing about the existence of a consistent choice set nor how it can be identified. The next section proves the existence, and provides an algorithm for identifying the maximal consistent choice set.

2.2 Farsightedly uncovered set

Given $B \subseteq A$, we say that $a \gg -$*covers* b in B if (i) $a, b \in B$, (ii) $a \gg b$, and (iii) $c \in B$ and $c \gg a$ implies $c \gg b$. More compactly, $a \gg -$covers b in B if $\{a\} \cup D(a) \cap B \subseteq D(b) \cap B$.

By the definition of indirect dominance, an alternative cannot indirectly dominate itself, i.e. $a \notin D(a)$. However, if $\{a\} \cup D(a) \cap B \subseteq D(b) \cap B$ and $\{b\} \cup D(b) \cap B \subseteq D(a) \cap B$, then $a \in D(a)$. Thus if $a \gg -$covers b in B, then b does not $\gg -$cover a in B.

Lemma 7 *The $\gg -$covering relation in B is asymmetric, for any $B \subseteq A$.*

Moreover, $\{a\} \cup D(a) \cap B \subseteq D(b) \cap B$ and $\{b\} \cup D(b) \cap B \subseteq D(c) \cap B$ imply $\{a\} \cup D(a) \cap B \subseteq D(c) \cap B$. Thus $a \gg -$covers b in B, and $b \gg -$covers c in B, then $a \gg -$covers c in B.

Lemma 8 *The $\gg -$covering relation in B is transitive, for any $B \subseteq A$.*

Denote by $UC(B)$ the $\gg -$*uncovered* set of B, i.e. the set of alternatives not $\gg -$covered in B by any element in B (cf. Fishburn, 1977; Miller, 1980). Since any transitive and asymmetric relation on a finite set has a maximal element, the $\gg -$uncovered set is nonempty for any nonempty B.

Lemma 9 *$UC(B)$ is nonempty if B is nonempty, for any $B \subseteq A$.*

We now strengthen of the $\gg -$uncovered set -concept. The *ultimate $\gg -$uncovered set* (cf. Miller, 1980; Dutta, 1988; Laslier, 1998) is defined recursively as follows. Set $UC^0 = A$, and let $UC^{k+1} = UC(UC^k)$, for all $k = 0, \ldots$. Then UC^∞ is the ultimate $\gg -$uncovered set. By Lemma 9, UC^k is nonempty for all k which implies that UC^∞ is nonempty.

Let $UC^\infty = Z$. By construction, no element in Z is $\gg -$covered in Z. Since A is nonempty and finite, we have, by Lemma 9, the following.

Lemma 10 *Z is nonempty.*

The aim of this subsection is to prove that a consistent choice set and, by Theorem 6, a stable set exists. We now show that the ultimate uncovered set is a consistent choice set.

Theorem 11 *Z is a consistent choice set.*

Proof. Z is nonempty by Lemma 10. Take $a \in Z$ and let $b \gg a$. We find an element in Z that indirectly dominates b but not a.

Since A is a finite set, there is a finite m such that $Z = UC^m$. Choose $b = c_0$ and, for all $j = 0, \ldots$, find k_j such that $c_{j+1} \gg -$covers c_j in UC^{k_j} and $c_j \in UC^{k_j} \setminus UC^{k_j+1}$. Let $j^* \leq m$ be the maximum number of such steps (see Figure 1, where $k_0 = 0, k_1 = 2$, and $k_2 = 3$). Then $c_{j^*} \in Z$.

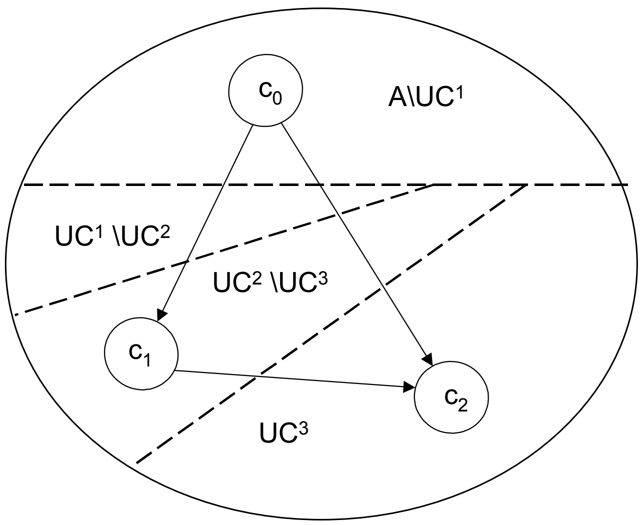

Figure 1.

Since $\{c_1\} \cup D(c_1) \cap UC^{k_0} \subseteq D(c_0) \cap UC^{k_0}$, and since $UC^{k_1} \subseteq UC^{k_0}$, it follows that $\{c_1\} \cup D(c_1) \cap UC^{k_1} \subseteq D(c_0) \cap UC^{k_1}$. As the same relation holds for c_1 and c_2, we have, by chaining the relations, that $\{c_2\} \cup D(c_2) \cap UC^{k_2} \subseteq D(c_0) \cap UC^{k_2}$ (depicted in Figure 1). By induction on $0, ..., j^*$, it follows that $\{c_{j^*}\} \cup D(c_{j^*}) \cap Z \subseteq D(c_0) \cap Z$. Thus $c_{j^*} \in D(c_0)$.

Choose $c_{j^*} = c$ (and recall that $c_0 = b$). Then $c \in D(b)$ and $c \in Z$. Since $b \in D(a)$ and $c \in D(b)$, we are done if $c \notin D(a)$. Suppose, to the contrary, that $c \in D(a)$. Since $a, c \in Z$, and $UC^{\infty} = Z$, it follows that $\{c\} \cup D(c) \cap Z \nsubseteq D(a) \cap Z$. Thus, since $c \in D(a)$, there is $d \in Z$ such that $d \in D(c) \setminus D(a)$. Since $\{c\} \cup D(c) \subseteq D(b)$, and $d \in D(c)$, we have that $d \in D(b)$. Thus $d \in D(b) \setminus D(a)$, as desired (see Figure 2).

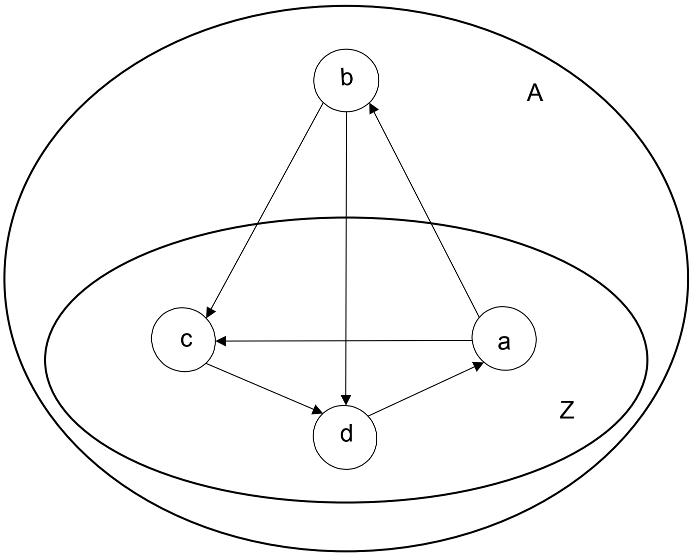

Figure 2.

■

The next result shows that Z is the (unique) maximal consistent choice set in the sense of set inclusion, given the asymmetric binary relation $\gg$.

Theorem 12 *Z is the maximal consistent choice set.*

Proof. Let X be a consistent choice set. We show that $X \subseteq Z$. Recall Definition 3: if $a \in X$ and $b \in D(a)$, then $X \cap D(b) \setminus D(a)$ is nonempty. Equivalently,

$$\{a\} \cup D(a) \cap X \nsubseteq D(b) \cap X, \quad \text{for all } a \in X, \text{ for all } b \in A. \tag{6}$$

By (6), $\{a\} \cup D(a) \nsubseteq D(b)$, for all $a \in X$. Thus, by the definition of covering in A, $X \subseteq UC(A)$. Thus, by (6), $\{a\} \cup D(a) \cap UC(A) \nsubseteq D(b) \cap$

$UC(A)$ for all $a \in X$. Thus, by the definition of covering in $UC(A)$, $X \subseteq UC(UC(A)) = UC^2$. By induction, $X \subseteq UC^\infty = Z$. ■

By Lemma 5 and Theorem 11, V^Z is a stable set. Moreover, by Theorem 11, the outcomes induced by V^Z are the maximal set of outcomes induced by any dynamic stable set.

Corollary 13 *V^Z is a stable set. Moreover, Z is the unique maximal set of outcomes that can be implemented in any dynamic stable set.*

Thus it is without loss of generality to focus on Z if one is interested interested on the welfare consequences of dynamically stable coalitional bargaining with farsighted agents.

3 Discussion

3.1 Relation to the largest consistent set

A consistent choice set meeting Definition 3 is more restrictive than Chwe's (1994) consistent set, defined as follows: A set $Z \subseteq A$ is consistent if $a \in Z$ and $a \rightarrow_S b$, then there is $c \in Z$ such that $c = b$ or $c \in D(a)$ and $c \not\succ_S a$.

Thus there are two weakenings relative to the definition of consistent choice set: (i) Chwe's consistent set only concerns the case where b directly dominates a. (ii) It requires that if b dominates a via $S \subseteq A$, then it is only S that does not benefit from c relative to a. A consistent choice set meeting Definition 3 makes both parts more stringent by requiring the condition to hold for all b that indirectly (and, hence, directly) dominate a, and that there is no S for which it holds. As a consequence, a consistent choice set meeting Definition 3 is consistent in the sense of Chwe, but not vice versa. Since the maximal consistent choice set, i.e. the ultimate uncovered set, is a consistent choice set in the sense of Chwe, his *largest* consistent set, which is the key solution of Chwe (1994), contains the ultimate uncovered set, defined with respect to the indirect dominance relation, as a subset (see

Figure 3).

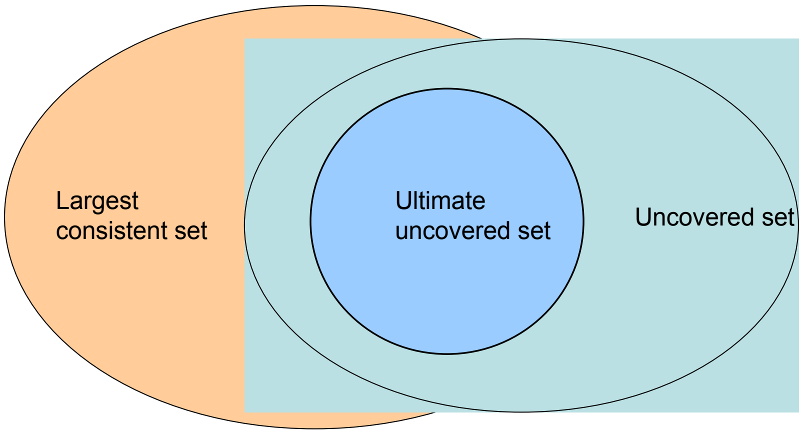

Figure 3.

3.2 Robustness of the model

As pointed out Xue (1998), farsighted dominance is not an unproblematic concept when it comes to capturing foresight of the agents. Namely, there is no guarantee that a deviating coalition in the middle of the deviation path finds it "optimal" to adhere to the expected action. It is not clear why the coalition would not choose another deviation, especially if this is in the end expected to bring all its members a higher payoff than the original one. Because of the implicit optimism embedded into the notion of indirect dominance, the largest consistent set of Chwe (1994) may be too inclusive.

This criticism is also valid also in the context of dynamic stable set. For if an active coalition cannot be sure that the projected outcome is eventually reached, it may not be in all the coalition members' interest to participate the coalition. But then an element outside the stable set need no longer be destabilized, invalidating the indirect dominance argument. Furthermore, if an element outside the stable set fails to be unstable, if an element outside the stable set is rendered stable, then there is no guarantee that no coalition will not challenge an outcome inside the stable set by demanding this newly stabilized outcome. Hence *also* internal stability may become invalidated..

However, in some scenarios the scepticism towards indirect dominance is not warranted. Optimism may not be needed to guarantee that the end of the deviation chain is reached. This is particularly true *when indirect dominance implies direct dominance* (the converse holds by definition). Then, by the definition of direct dominance, the first deviating coalition can reach the final element in the deviation path in a single step. Thus there is no need to worry about the credibility of the interim deviations. Moreover, since the dynamic stable set is defined with respect to indirect dominance, it is still true that any outcome outside the stable set is directly dominated by an outcome in the set. Thus a deviation from the stable set leads back, in one step, to the stable set. By the definition of indirect dominance, the latter outcome is *not* preferred by all the members of the originally deviating coalition and hence the Harsnyi-critique is avoided. Both internal and external stability should hold without reservations.

While asking indirect dominance to coincide with direct dominance is a strong requirement, it holds in a natural class of coalitional games. We apply the dynamic stable set -solution to an *unbounded agenda formation* -problem, where a subsets of agents (with certain cardinality) are allowed to challenge the status quo with *any* alternative.

4 Unbounded agenda formation

Agenda setting has been viewed as a major problem in voting theory. Moulin (1979), Shepsle and Weingast, (1984), and Banks (1985) analyse majority voting within a *finite* agenda, i.e. binary tree. From these studies it is clear that the agenda setter can influence the implemented outcome. Dutta, Jackson and Le Breton (2001a, see also 2001b, 2002) study endogenous agenda formation under the hypothesis that the length of an agenda is finite. We, on the contrary, allow *unbounded* formation of the agenda.

Let agents $N = \{1, ..., n\}$ engage in the process of making a a decision over the set A of alternatives. For any $k \in \{1, ..., n\}$, the *agenda setting game* for the $k-$majority rule where, if $a \in A$ is the status quo, then the coalition S can induce b as the status quo if the size of S is at least k, i.e., for any $a \in A$, $a \rightarrow_S b$ for any $b \in A$ if $|S| > k$. The intuition is that if the status quo a is not challenged by any coalition, then it is implemented.

Let the irreflexive binary relation M^k denote the $k-$majority dominance over pairs of A, i.e. aM^kb if and only if there is a coalition S with at least k members such that $a \succ_S b$. Since in this set up, $\rightarrow_S$ applies to all pairs in $A \times A$ whenever S contains at least k members, indirect dominance equals direct dominance. Denote by $D^k(a)$ the outcomes that (directly or indirectly) M^k-dominate a.

By Theorem 12, the ultimate uncovered set defined with respect to M^k-relation completely characterizes the implementable outcomes. If $k = (n+1)/2$, and n is odd, then the M^k-relation is a tournament, the case studied e.g. by Dutta (1988). The ultimate uncovered set is his *covering set*.[7] Hence the largest dynamic stable set of commitment free agenda setting procedure with majority voting gives an interpretation to the covering set, called for by Dutta (1988).

The following result demonstrates that the outcomes that are implementable via the agenda setting game are always Pareto-optimal if only

[7] In fact, any consistent choice set is a covering set, see Vartiainen (2006).

coalitions consisting a majority can upset the status quo. Hence the "commitment-free" agenda setting leads to efficiency, as suggested by the string version of the *Coase theorem*, if one does not allow too much "diversity of opinions" in the agenda formation.

Proposition 14 *If $k \geq n/2$, then all outcomes that are implementable in a dynamic stable set V of the agenda setting game for the $k-$majority rule are Pareto-optimal.*

Proof. By Lemma 4, $\{a' : (h, a') \in V\}$ is a consistent choice set. Suppose that $b \succeq_N a$ for some $a \in \{a' : (h, a') \in V\}$. Then $bM^k a$. By Definition 3, there is c such that $c \in D^k(b) \setminus D^k(a)$. Take any S such that $c \succ_S b$ and $|S| \geq k$. By the transitivity of $\succeq_i$'s, $\{j : c \succ_j a\} \supseteq S \cap N$. But then, since $k \geq 2/n$, $c \in D^k(b)$ implies $c \in D^k(a)$, a contradiction. ■

However, when also coalitions smaller than the majority are allowed to challenge the status quo, Pareto-optimality need not prevail. To see this, choose $n = 3$ and $k = 1$, and let the agents' preferences over $\{a, b, c\}$ be

$$\begin{aligned} 1 &: a \succ b \succ c, \\ 2 &: a \succ c \succ b, \\ 3 &: c \succ a \succ b. \end{aligned}$$

Then no outcome is M^1-covered. Thus also b, which is Pareto-dominated by a, belongs to the (ultimate) uncovered set and, hence, is implementable in a dynamic stable set.

We conclude that unless the agenda control is given to a majority, there is no guarantee that the outcome will be Pareto-optimal. Hence, the strong version of the Coase theorem fails.

5 Conclusion

One of the key problems with the von Neumann-Morgenstern stable set solution has been the existence problem in general contexts: In a domination-rich environment the solution tends not to exists (see e.g. Shubik, 1997). Another problem is that the solution - when it exists - is not easily computable. For these reasons and others, many less demanding solutions have been developed (see e.g. Rubinstein, 1980; Dutta, 1988; and Moulin, 1986. Greenberg's, 1990, offers a general approach).

We argue that the existence problem stems from an implicit stationarity assumption: Coalitional strategies are dependent only on the status quo outcome. We show that if history dependent strategies are allowed, and agents are farsighted, then a (dynamic) stable set always exists. Moreover, we establish an algorithm that produces the (unique) maximal stable set.

The algorithm is a version of the iterative procedure that identifies the ultimate uncovered set in the standard social choice setting.

The biggest problem with our suggested solution is not related to the dynamicity but rather the way farsightedness is modeled. While the indirect dominance -relation is commonly used in the literature (see e.g. Harsanyi, 1974; Chwe, 1994; Diamantoudi and Xue, 2005, 2006; Xue, 1998; Page et al., 2005), it suffers from incredibility: It assumes that the agents conform with a presupposed deviation path without considering their strategic options in the middle of the path (see Xue, 1998, for analysis).

However, in the particular case when the a permissible coalition can always induce any outcome, i.e. indirect dominance implies direct dominance, our solution is free from these considerations. The unbounded agenda formation -game falls into this category. In this game coalitions of at least certain size are allowed to challenge the outcome on the table without a unboundedly many times.

We show that when the coalitions are required to form a majority, all the implementable outcomes are Pareto-optimal. However, with smaller coalition sizes, also inefficient outcomes may become implemented. Since the underlying process can be interpreted to reflect genuine non-commitment from the party of the coalitions, this suggests that the Coase theorem is valid even under nonstationary strategies (cf. Gomez and Jehiel, 2005; Konishi and Ray, 2003) too much "diversity" is not permitted in valid coalition formations

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